Logic, Language and Modularity[1]

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Some very basic questions (as basic as you can get):

1. What is logic?
2. What is the relationship between language and logic?
3. How do we distinguish logical from non-logical inference?

Response to these questions within modern/mathematical logic:

1. Logic is a system that a logician constructs: a system which includes axioms and rules of inference (a logical syntax).
2. There is no logic without language: the first step in developing a logical system is defining a language on which one can define the logical syntax.
3. A designated set of axioms and rules of inference are called *logical*, whereas other axioms are called *non-logical* (so called, *theories*). There might be constraints on what can be called logical, but the dividing line is more or less arbitrary.

But this is not a response to the questions, if understood as questions about human cognition, i.e., questions about *the roots of human knowledge*:

1. **Logic**:
   What is human logic (*natural logic* in the sense of e.g., Lakoff 1970)?
2. **Language**:
   What is the relationship between natural language and natural logic?
3. **Modularity**:
   How do we distinguish inference derived by natural logic from inference derived with the help of other cognitive systems?

General Claims:

1. **Logic**
   Human Logic is partly accounted for by the workings of a single cognitive system (The Deductive System, DS)
2. **Language**
   DS is a component of the linguistic system.
3. **Modularity**
   DS is a modular system which derives certain logical consequences from sentences based on *formal properties alone*.

[1] Almost all of the substantial material discussed here is based on joint work Martin Hackl (*Linguistics and Philosophy* 2006)

Specific Claims

1. **Certain aspects of arithmetic are part of *natural logic***: DS derives inferences pertaining to degrees and scales. These inferences might be crucial building blocks for our arithmetic abilities.

2. **Density**: However, the relevant notion of degree is characterized by the axioms of densely ordered domains. Cardinality (natural number) does not appear to be a notion of DS.

3. **Modularity:** Cardinality does contribute to meaning, but this contribution results from extra-linguistic interactions.

1. Older Evidence for DS

Unfortunately, not enough time, but some of the key observations can be summarized as follows:

There are rules of grammar that are sensitive to patterns of Logical Inference.

1. Polarity Licensing: Rules pertaining to the licensing of polarity items (Fauconnier 1975, 1979, Ladusaw 1979,Kadmon and Landman 1993, and quite a bit of subsequent work)
2. Scope Economy: Rule that imposes constraints on the scope of quantifiers (Fox 1995, 2000), Reinhart (2006), Spector and Mayr (in press), based on the logical properties of the environments in which they occur (very much inspired by ideas in Reinhart 1983, and Grodzinsky and Reinhart 1992).
3. *Contradiction: A general constraint against logical contradictions and tautologies (Barwise and Cooper 1983, Chierchia 1984, 2005, von Fintel 1993, Gajewski 2002, passim)

And here is one example (of *contradiction) (von Fintel 1993):

(1) a. Every man but John came to the party.

   1. It is false that every man came to the party.
   2. It is true that every man other than John came to the party.

   b. *A man but John came to the party.
   Conveys:
   a. It is false that a man came to the party,
   b. It is true that a man other than John came to the party.

2. Degree Constructions

(2) a. John has more than 3 children.
   $\exists n>3$: John has n children.
   b. John is more than 6 feet tall.
   $\exists d>6$: John is d feet tall.

Often Postulated:

A. Scales (various domains of degrees/numbers)
B. A notion of measurement: functions from various objects to various scales
C. Two types of scales, and hence two different notions of measurement:
   1. Various expressions that combine with count nouns (*3 girls, more than three girls, how many boys…*) require the notion of cardinality/counting for their understanding, i.e. a function from sets to discrete scales.
   2. Other degree expressions (*6 feet tall, taller than 6 feet, how tall, how much water…*) rely on different scales, perhaps on dense scales, something closer to the real or rational numbers.

Goals of this talk:

1. To give you a quick taste of an argument from Fox and Hackl (2006) that degree/measurement scales are always dense (based on *contradiction):

(3) The Universal Density of Measurements (UDM): Measurement Scales that are needed for Natural Language Semantics are *always* dense.

In other words, an argument for two claims:

(4) a. The Intuitive Claim: Scales of height, size, speed, and the like are dense.
    b. The Radical Claim: All Scales are dense; cardinality in not a concept of NLS.

2. To explain why the Radical Claim must be accompanied by a strong modularity thesis, and to give you a taste of some of the independent motivation for this modularity thesis.

3. A constraint on *only* and on exhaustive meanings

Background: Simple implicatures and their correspondence to sentences with *only*

(5) a. John has three children.
    Implicature: John has exactly three children.
    b. John has very few children. He only has THREE.

(6) a. John weighs 150 pounds.
    Implicature: John weighs exactly 150 pounds.
    b. John weighs very little. He only weighs 150.

3.1. Density as an intuitive property of scales

THE BASIC EFFECT

(7)a. John weighs more than 150 pounds.
    *Implicature: There is no degree great than 150, d, s.t. John weighs d pounds.
    I.e., John weighs exactly S(150) pounds. Where S is the successor function
    b. John weighs very little. *He only weighs more than 150.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(8) a. You're required to weigh more than 300 pounds (if you want to participate in this fight).
Implicature: There is no degree greater than 300, d, s.t. you are required to weigh more than d pounds.
b. You're only required to weigh more than 300 pounds.

EXISTENTIAL MODALS DO NOT

(9) a. You're allowed to weigh more than 150 pounds (and still participate in this fight).
*Implicature: There is no degree greater than 150, d, s.t. you are allowed to weigh more than d pounds.
b. *You're only allowed to weigh more than 150 pounds.

3.2. Density as a formal property

THE BASIC EFFECT

(10) a. John has more than 3 children.
*Implicature: John has exactly 4 children.
b. John has very few children. *He only has more than THREE.

UNIVERSAL MODALS CIRCUMVENT THE PROBLEM

(11) a. You're required to read more than 30 books.
Implicature: There is no degree greater than 30, d, s.t. you are required to read more than d books.
b. You're only required to read more than 30_F books.

EXISTENTIAL MODALS DO NOT

(12) a. You're allowed to smoke more than 30 cigarettes.
*Implicature: There is no degree greater than 30, d, s.t. you are allowed to smoke more than d cigarettes.
b. *You're only allowed to smoke more than 30_F cigarettes.

Note: My goal here was only to give you a flavor of the argument. The argument, itself, is based on a much larger paradigm, and in order to evaluate it, you will have to consult the relevant literature. In particular: Fox and Hackl (2006, *Linguistics and Philosophy*), Fox (2007, *SALT Proceedings*), Nouwen (2008, *Natural Language Semantics*), as well as an alternative perspective on part of the paradigm developed in Abrusan and Spector (2008, in press *Journal of Semantics*).

4. Modularity

4.1. A Problem:

(13) a. I can say with certainty that John has more than 3 children.
Implicature: I can only say with certainty that John has more than 3_F children.
b. I can only say with certainty that John has more than 3 children.

The truth conditions of these sentences, suggest that only integers count.

4.2. A restatement of the problem (towards a solution)

There is a more basic problem: the rounding/granularity problem.

(14) John is six feet tall

The meaning of a sentence is determined in a context which specifies (among many other things) a level of granularity, G. We can think of G as an equivalence relation.

(15) *John is exactly six feet tall* expresses in a context C, the claim that John's height stands in the G relation to the degree *six-feet*.

In short: $\text{Height}_{\text{feet}}(J) \in \text{Equivalence-Class}_G(6)$

(16) *John is exactly 15 years old* expresses in a context C, the claim that John's age (in years) Age(J) stands in the G-relation to 15.

If, $G = \{\langle x,y\rangle$: there is a natural number n, s.t. $x \in [n, n+1)$ and $y \in [n, n+1)\}$, then *John is (exactly) 15 years old* will express the claim that $\text{Age}_{\text{years}}(J) \in [15, 16)$

In short: $\text{Age}_{\text{years}}(J) \in [15, 16)$

What we wanted to say:

(17) Only[John is more than 15_F years old]
Expresses the claim that:
i. $\text{Age}_{\text{years}}(J) > 15$
ii. $\forall d\ [\text{Age}_{\text{years}}(J) > d] \rightarrow [15 \geq d]$.

The truth conditions can never be met since the set of degrees is dense.

However, this line of reasoning ignores contextual parameters, and in particular, the granularity parameter G.

Problem: It turns out that once G is taken into account, it is no longer obvious that the truth-conditions are contradictory:

(18) Only[John is more than 15_F years old]
Expresses the claim that:
i. $\text{Age}_{\text{years}}(J) > [15, 16)$
ii. $\forall EC\ [\text{Age}_{\text{years}}(J) > EC] \rightarrow [[15, 16) \geq EC]$.
(where EC ranges over equivalence classes determined by G)

This is not contradictory.

4.3. Modularity

G doesn't enter the picture at the level at which *Contradiction is evaluated, though it does enter when the truth of a sentence is evaluated in a particular context.

We thus postulate a deductive system, DS, (Fox 2000, Gajewski 2002/2003) in which sentences are evaluated and ruled out if they are can be proven to be unusable (contradictions, tautologies, etc.).

In DS, sentences are ruled out if they can be proven to be contradictory (under the stringent granularity, =).

Once a sentence passes DS, it is evaluated in a particular context, where a more lax level of granularity may affect the interpretation.

4.4. Other reasons for a modular system (following the logic of Gajewski 2002)

von Fintel 1993:

(19) a. Every man but John arrived.
It's not true that every man arrived, yet it is true that every man other than John arrived.
b. *A man but John arrived.
It's not true that a man arrived, yet it is true that a man other than John arrived.

Dowty 1979:

(20) *John accomplished his mission for an hour.
There is a time interval in the past T s.t. Length(T) = one hour and
$\forall t \subseteq T$ John accomplished his mission in t.

And many, many other examples. See, e.g. , Chierchia 2005, and Menéndez-Benito 2005.

But, we seem to know what to do with certain contradictions.

(21) a. This table is both red and not red.
b. He's an idiot and he isn't.
c. I have a female (for a) father.
d. I have 3.5 children.

(22) What you're saying is obviously false.
a. It fallows that there is no man who arrived and yet that a man other than John arrived.
b. #It fallows that a man but John arrived.

Gajewski (2002): Nevertheless there is a general condition that disapproves of contradictions.
But the relevant system (DS) is modular: it is blind to the non-logical vocabulary.

4.5. Back to the UDM

If DS is thought of in syntactical terms (the terms of logical syntax), then our goal would be to spell out axioms and rules of inference as possible theories of DS.

If the arguments for the UDM are correct, the following must be theorems of DS.

(23) a. Universal Density: $\forall d_1, d_2\ [d_1 > d_2 \rightarrow \exists d_3(d_1 > d_3 > d_2)]$
b. Lexical Monotonicity: lexical n-place relations that have a degree argument are upward monotone.
c. Lexical Closed Intervals: if R is a lexical n-place relation, whose m^{th} argument is a degree, then for every *w*, and for every $x_1,\ldots,x_{n-1}$ $Max_{inf}(\lambda d.R(x_1)\ldots (d) \ldots(x_{n-1}))(w)$ is defined.
d. Commutativity: Two existential quantifiers can be commuted.[2]

5. Syntactic Contextual Restriction

Kroch (1989): When the context provides an explicit set of alternatives, negative islands are circumvented:

(24) Among the following, please tell me how many points Iverson didn't score?
a. 20 b. 30 c. 40 d. 50

What is the most informative degree in C, s.t. Iverson didn't score d points?
C = {20, 30, 40, 50}

This extends to implicatures and *only*.
(25) Iverson sometimes scores more than 30 points. But today he only scored more than 20_F.

(26) A: How many points did Iverson score last night?
B: I don't know.
A: Was it more than 10, more than 20 or more than 30.
B: He scored more than 20 points
Implicature: he didn't score more than 20.

Exh/Only[C] [Iverson scored more than 20 points]
C= {that Iverson scored more than 30 points, that I. scored more than 30 points}

6. Conclusions

1. There is evidence that grammar rules out formal contradictions. If this is correct, it could be used to identify the formal vocabulary of natural language, and the rules of logical-syntax that characterize this vocabulary.
2. The semantics for the logical rules that characterize degree expressions invovles dense degree domains.

[2] (23)d is also crucial to deliver the results needed for the proposal in Fox (2000).

3. Scalar implicatures are derived by a lexical item, a member of the logical vocabulary, *exh*.
4. Integers enter into the determination of truth conditions. However, this takes place within pragmatic system, via contextual parameters (integers are not part of what is sometimes called *narrow content*).
5. There are two different ways in which context can affect truth conditions with very similar surface consequences: contextual parameters and covert pronouns.
6. There is a possible connection with work in experimental psychology which aims to discover the origins of human reasoning about quantities and numerosities. (Carey, Dehaene, Gelman and Gallistel, Spelke, among many others.) Recurring hypothesis: Prior to the development of adult arithmetic there is a core system that allows the measurement (or at least the estimation) of quantities, but crucially does not have access to anything like the notion of a natural number. Perhaps the core system is the one relevant for NLS.