Only a little bit more

1. Proposed entries for *only/exh*

Horn-based

(1) $only(x_\alpha)(A_{\alpha t}) \Leftrightarrow A(x) \,\&\, \forall y[A(y) \rightarrow y \leq x]$

(2) $only(x_\alpha)(A_{\alpha,st})(w) \Leftrightarrow A(x)(w) \,\&\, \forall y[A(y)(w) \rightarrow (A(x) \Rightarrow A(y))]$

G&S

(3) $only(Q_{\alpha t,t})(A_{\alpha t}) \Leftrightarrow Q(A) \,\&\, \neg\exists B_{\alpha t}\, [Q(B) \,\&\, (B \subset A)]$

2. Only and QPs (The Advantage of G&S)

(4) only (a-boy)(came) $\Leftrightarrow$ Exactly one person came and that person was a boy.

(5) only(J or B)(came) $\Leftrightarrow$ Exactly one person came and that person was either J or B.

(6) only (two boys)(came) $\Leftrightarrow$ Exactly two people came and those people were boys.

3. Other Options (based on Rooth's syntax)

(7) $only(A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w)$ & there is exactly one proposition in A that entails p.
where A is going to be the Hamblin-denotation of the relevant *wh*-question.

Alternatively:

Spector (2004):
A = closure under $\{\wedge,\vee\}$ of the Hamblin denotation of the question.

(8) Sauerland-Spector-based:
$only(A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w) \,\&\, \forall q \in Str.A(p,A)$
$[\neg\exists q' \in Str.A(p,A)[p \wedge \neg q \Rightarrow q']] \rightarrow q(w) = 0$

(9) S&S-based:
$only(A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w) \,\&\, \forall q \in A$
$[K\neg q$ is consistent with $Kp \wedge PI(A)(p)] \rightarrow q(w) = 0$

$PI(A)(p) = \cap\{\neg Kq: q \in A$ and $q \Rightarrow p$ asymmetrically$\}$, or alternatively
$PI(A)(p) = \cap\{\neg Kq: q \in A$ and $\neg Kq$ is consistent with $Kp\}$

3. vR&S and the “functionality problem”: did G&S loose the advantage? [1]

(10)Who came to the party?

Possible answers:

a. Two boys c. John or Bill

b. At least two boys d. John or Bill or both

(11)a. ∃f Only(f(two-boys))(came)

b. ∃f Only(f(at-least-two-boys))(came)

c. ∃f Only(f(John or Bill))(came)

d. ∃f Only(f([John or Bill] or both))(came)

vR&S provide us with an analysis of sentences in which *only* appears next to an existential QP which can work with any of the lexical entries proposed.

So is there still an advantage for G&S over any of the alternatives?

In fact, could we turn the argument on its head? The Horn-based analysis predicts that the quantifiers will not be good arguments of *only*. Hence we have to resort to the CF analysis (or the Heim/Kamp alternative that vR&S implement). Under G&S, we would have to claim that the CF analysis is the only one available. Any reason to doubt this claim might serve to argue in favor of the Horn-based analysis.

4. A possible complication

(12) How many people came to the party?

Possible answers:

a. Two Only(two)(λd. d many people came to the party)

b. At least two Only(at least two)(λd. d many people came to the party)

(13) How tall is John?

Possible answers:

a. 6 feet Only(six feet)(λd. John is d tall)

b. At least 6 feet Only(at least six feet)(λd. John is d tall)

(14) a. 6-feet' = 6F

a'. 6-feet' $=\lambda P_{dt}.P(6F)$

b. at-least-6-feet' = $\lambda P_{dt}.\exists d\geq 6(P(d))$

(15) $Only_{GS}(6F) = \{\{6F\}\}$

[1] The formulation of vR&S that I provide here is due to Heim (class notes 2004).

This, of course, suffers from the problem G&S had with distributive predicates.
So, let's move to vR&S's alternative:

(16) $only(Q_{\alpha t,t})(A_{<s,\alpha t>})(w) \Leftrightarrow Q(A(w))$ &
$\neg\exists B_{\alpha t}$ $[[Q(B)$ & $(B \subset A(w))$ & $\exists w'(Q(A(w'))$ & $A(w') = B)]]$

(17) $only_{vR\&S}(6F)(\lambda w'. \lambda d.$ John is d tall in w')(w) $\Leftrightarrow$
John is 6F tall in w & there is no proper subset of {d: John is d tall in w}, B, such that $6F \in B$ and it is possible that {d: John is d tall} = B. $\Leftrightarrow$
{d: John is d tall in w} = {d: $d \leq 6F$}

But then (as far as I can see):

(18) $only_{vR\&S}$(at-least-6F)(λw'. λd. John is d tall in w')(w) $\Leftrightarrow$
John is 6F tall in w & there is no proper subset of {d: John is d tall in w}, B, such that $6F \in B$ and it is possible that {d: John is d tall} = B. $\Leftrightarrow$
{d: John is d tall in w} = {d: $d \leq 6F$}

Perhaps:
(19) at-least-6-feet' = {d: $d \geq 6F$}

(20) How tall is John?
At least 6 feet $\exists$fOnly(f(at least six feet))(λd. John is d tall)

Question: Are there any tests for such an analysis, e.g. discourse anaphora?

(21) a. John read at least 2 books. I also read them.
b. John is at least 6 feet tall. I am also that tall.

(22) Who came to the party?
At least John and Bill. They seemed to have a good time.
(Irene Heim, p.c.)

4. A different approach to *at least*

At least and *at most* are focus sensitive operators (Krifka 1999).
They also seem to have superlative morphology (a point stressed by M. Hackl, p.c., for a very long time. C.f., Nouwen *SuB* 2004)

(23) John at least read TWO books
2 is the minimal number such that it is possible (given the speakers belief state) that John read exactly that number of books.

(24) John at most read TWO books

2 is the maximal number such that it is possible (given the speakers belief state) that John read exactly that number of books.

(25)a. [[at least (A)(p)(w)]] = 1 iff the world, $w' \in E_w$, in which the fewest propositions in A are true is s.t. exh(A)(p)(w) = 1.

b. [[at most (A)(p)(w)]] = 1 iff the world, $w' \in E_w$, in which the most propositions in A are true is s.t. exh(A)(p)(w) = 1.

When *at least* appears unembedded, E_w is taken to be the set of worlds consistent with the speakers beliefs.

4.1. The implicature in question is not consistent with superlative semantics

(26) a. #You are the best mother I have.
b. You are the best mother around.
b. You are the best mother I could ask for.

(27) $est'(A_{<\alpha,t>}) = \lambda H_{<\alpha,dt>}: \exists xy \in A$ y is more H than x.
$\{x \in A: \neg\exists y \in A$ y is more H than x$\}$

(28) [[at most (A)(p)(w)]] = Exh(A)(p)(the'(est'(E_w)(H_A))

H_A(w)(d) = d propositions in A are true in w.

(29) [[at least (A)(p)(w)]] = Exh(A)(p)(the'(est'(E_w)($H^{less}{}_A$))

$H^{less}{}_A$(w)(d) = d-few propositions in A are true in w.

Why is the implicature in question not present?

Intuition:
Presuppositions of *at least/at most*: E_w must contain two worlds in which the number of true propositions in A is different. Hence the content of a sentence in which *at least* has matrix scope can't determine how many proposition in A are true.

Implementation:

(30) John is at least 6 feet tall

(31) Exh(A)(at least(A') John is $6F_F$ tall)(w) = 1 iff
at least(A') John is $6F_F$ tall (w) = 1 and
$\forall d \neq 6$ (at least(A') John is dF_F tall (w) = 0) iff
at least(A') John is $6F_F$ tall (w) = 1

4.2. Possible Advantage

Iatridou, Anagnostopoulou, and Izvorski (2001), Von Fintel and Iatridou (2002)

(32) a. John has been in Paris since July. (both U and E readings)
b. John has been in Paris at least since July. (only U reading)

If the UDM is correct, as well as the lexical entries above, we predict that a*t least* and *at most* just like *only* will be inconsistent with N-open properties:

(33) a. I am at least sure that he's been in Paris since JULY.
(34) a. John has been in Paris at most since July. (only U reading)
b. I can at most guarantee that he's been in Paris since July.

(35) a. * John at least has not smoked 30_F cigarettes.
b. * John at most has not smoked 30_F cigarettes.
Benjamin Spector (p.c.)

(36) a. John is at least not allowed to smoke 30_F cigarettes.
b. John is at most not allowed to smoke 30_F cigarettes.

5. A different approach to (*p or q) or both*

Hurford's Generalization: *A or B* is infelicitous when B entails A.[2]

(37) a. ??John is an American or a Californian.
b. ??I was born in France or Paris.

Hurford used this generalization to argue for a strong meaning for disjunction (ExOR):

(38) I will apply to Cornell or UMASS or to both.

We can extend this argument to other scalar items:

(39) a. I will read two books or three.
b. I will do some of the homework or all of it.

(38') Exh(C)(I will apply to Cornell or_F UMASS) or [I will apply to Cornell and UMASS]

What happens if we add yet another exhaustive operator? Nothing.
What happens if we apply Sauerland's algorithm? No secondary implicatures.

[2] See also Simons, M. (2000). Issues in the Semantics and Pragmatics of Disjunction. New York and London, Garland Pub.

6. Remaining Implicatures:

(40) Who came to the party?
2 or more boys.
Implicature: Girls didn't come.
Only 2 or more boys. (entails: girls didn't come)

(41) Only(A)(2 or more boys)(w)???

(42) Who came to the party?
2 boys.
Only 2 boys.
Implicature: exactly 2 boys came and Girls didn't come.

Spector (2004):
A = closure under $\{\wedge,\vee\}$ of the Hamblin denotation of the question.

(9) S&S-based:
$\text{only}(A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w)\ \&\ \forall q \in A$
$[K\neg q \text{ is consistent with } Kp \wedge PI(A)(p)] \rightarrow q(w) = 0$

$PI(A)(p) = \cap\{\neg Kq: q \in A \text{ and } q \Rightarrow p\}$

Assume the following domain: b1, b2, b3 g1,g2

(43) Only(A)(2 boys came to the party)(w) ⇔
2 boys came to the party in w and
not (((b1 & b2) or (b1 & b3) or (b2 & b3)) and g1) and
not (((b1 & b2) or (b1 & b3) or (b2 & b3)) and g2) and
not (b1 & b2 & b3)

(44) Who came to the party?
At least 2 boys.
Implicature ??? Girls didn't come.
*Only at least 2 boys

If there is an implicature, we might consider the following:

(9') $\text{only}(A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w)\ \&\ \forall q \in A$
$[K\neg q \text{ is consistent with } Kp \wedge PI(A)(p) \wedge \text{Presuppositions}(p)]$
$\rightarrow q(w) = 0$