Only a little bit more

1. Proposed entries for only/exh

Horn-based

(1) only(x,α)(Aαt) ⇔ A(x) & ∀y[A(y) → y ≤ x]

(2) only(x,α)(Aαst)(w) ⇔ A(x)(w) & ∀y[A(y)(w) → (A(x) ⇒ A(y))]

G&S

(3) only(Qαt,t)(Aαt) ⇔ Q(A) & ¬∃Bαt [Q(B) & (B ⊂ A)]

2. Only and QPs (The Advantage of G&S)

(4) only (a-boy)(came) ⇔ Exactly one person came and that person was a boy.

(5) only(J or B)(came) ⇔ Exactly one person came and that person was either J or B.

(6) only (two boys)(came) ⇔ Exactly two people came and those people were boys.

3. Other Options (based on Rooth’s syntax)

(7) only(A<st,t>)(pst)(w) ⇔ p(w) & there is exactly one proposition in A that entails p.

where A is going to be the Hamblin-denotation of the relevant wh-question.

Alternatively:

Spector (2004):
A = closure under {∧, ∨} of the Hamblin denotation of the question.

(8) Sauerland-Spector-based:

only(A<st,t>)(pst)(w) ⇔ p(w) & ∀q ∈ Str.A(p,A)

[¬∃q’ ∈ Str.A(p,A)[p ∧ ¬q ⇒ q’]] ⇒ q(w) = 0

(9) S&S-based:

only(A<st,t>)(pst)(w) ⇔ p(w) & ∀q ∈ A

[K¬q is consistent with Kp & PI(A)(p)] ⇒ q(w) = 0

PI(A)(p) = ∩{¬Kq: q ∈ A and q⇒p asymmetrically}, or alternatively

PI(A)(p) = ∩{¬Kq: q ∈ A and ¬Kq is consistent with Kp}
3. vR&S and the “functionality problem”: did G&S lose the advantage? ¹

(10) Who came to the party?
   Possible answers:
   a. Two boys          c. John or Bill
   b. At least two boys  d. John or Bill or both

(11) a. \( \exists f \text{ Only}(f(\text{two-boys}))(\text{came}) \)
    b. \( \exists f \text{ Only}(f(\text{at-least-two-boys}))(\text{came}) \)
    c. \( \exists f \text{ Only}(f(\text{John or Bill}))(\text{came}) \)
    d. \( \exists f \text{ Only}(f(\text{[John or Bill] or both}))(\text{came}) \)

vR&S provide us with an analysis of sentences in which only appears next to an existential QP which can work with any of the lexical entries proposed.

So is there still an advantage for G&S over any of the alternatives?

In fact, could we turn the argument on its head? The Horn-based analysis predicts that the quantifiers will not be good arguments of only. Hence we have to resort to the CF analysis (or the Heim/Kamp alternative that vR&S implement). Under G&S, we would have to claim that the CF analysis is the only one available. Any reason to doubt this claim might serve to argue in favor of the Horn-based analysis.

4. A possible complication

(12) How many people came to the party?
   Possible answers:
   a. Two \( (\lambda d. \text{d many people came to the party}) \)
   b. At least two \( (\lambda d. \text{d many people came to the party}) \)

(13) How tall is John?
   Possible answers:
   a. 6 feet \( (\lambda d. \text{John is d tall}) \)
   b. At least 6 feet \( (\lambda d. \text{John is d tall}) \)

(14) a. 6-feet' = 6F
    a'. 6-feet' = \( \lambda P_{dt}. P(6F) \)
    b. at-least-6-feet' = \( \lambda P_{dt}. \exists d \geq 6(P(d)) \)

(15) \( \text{Only}_{GS}(6F) = \{ \{6F\} \} \)

¹ The formulation of vR&S that I provide here is due to Heim (class notes 2004).
This, of course, suffers from the problem G&S had with distributive predicates. So, let’s move to vR&S’s alternative:

(16) \[ \text{only}(Q_{at,t})(A_{\text{at}-})_{t}(w) \Leftrightarrow Q(A(w)) \land \neg \exists B_{at} \left[ [Q(B) \land (B \subset A(w)) \land \exists w'(Q(A(w')) \land A(w') = B)] \right] \]

(17) \[ \text{only}_{\text{vR&S}}(6F)(\lambda w'. \lambda d. \text{John is } d \text{ tall in } w')(w) \Leftrightarrow \]
John is 6F tall in w & there is no proper subset of \{d: John is d tall in w\}, B, such that 6F \in B and it is possible that \{d: John is d tall\} = B. \Leftrightarrow
\{d: John is d tall in w\} = \{d: d \leq 6F\}

But then (as far as I can see):

(18) \[ \text{only}_{\text{vR&S}}(\text{at-least-6F})(\lambda w'. \lambda d. \text{John is } d \text{ tall in } w')(w) \Leftrightarrow \]
John is 6F tall in w & there is no proper subset of \{d: John is d tall in w\}, B, such that 6F \in B and it is possible that \{d: John is d tall\} = B. \Leftrightarrow
\{d: John is d tall in w\} = \{d: d \leq 6F\}

Perhaps:

(19) \text{at-least-6-feet' } = \{d: d \geq 6F\}

(20) How tall is John?
At least 6 feet \[ \exists f \text{Only}(f(\text{at least six feet})(\lambda d. \text{John is } d \text{ tall})) \]

Question: Are there any tests for such an analysis, e.g. discourse anaphora?

a. John read at least 2 books. I also read them.
b. John is at least 6 feet tall. I am also that tall.

(22) Who came to the party?
At least John and Bill. They seemed to have a good time.
(Irene Heim, p.c.)

4. A different approach to at least

At least and at most are focus sensitive operators (Krifka 1999). They also seem to have superlative morphology (a point stressed by M. Hackl, p.c., for a very long time. C.f., Nouwen SuB 2004)

(23) John at least read TWO books
2 is the minimal number such that it is possible (given the speakers belief state) that John read exactly that number of books.

(24) John at most read TWO books
2 is the maximal number such that it is possible (given the speakers belief state) that John read exactly that number of books.

(25)a. \[[\text{at least } (A)(p)(w)]\] = 1 iff the world, \(w' \in E_w\), in which the fewest propositions in A are true is s.t. \(\text{exh}(A)(p)(w) = 1\).

b. \[[\text{at most } (A)(p)(w)]\] = 1 iff the world, \(w' \in E_w\), in which the most propositions in A are true is s.t. \(\text{exh}(A)(p)(w) = 1\).

When \textit{at least} appears unembedded, \(E_w\) is taken to be the set of worlds consistent with the speakers beliefs.

4.1. The implicature in question is not consistent with superlative semantics

(26) a. \#You are the best mother I have.
   b. You are the best mother around.
   b. You are the best mother I could ask for.

(27) \(\text{est}'(A_{<\alpha,\beta}) = \lambda H_{<\alpha,\beta}: \exists x y \in A \ y \text{ is more } H \text{ than } x.\)

\[
\{ x \in A : \neg \exists y \in A \ y \text{ is more } H \text{ than } x \}
\]

(28) \[[\text{at most } (A)(p)(w)]\] = \(\text{Exh}(A)(p)(\text{the}'(\text{est}'(E_w)(H_{A})))\)

\(H_A(w)(d) = d \text{ propositions in } A \text{ are true in } w.\)

(29) \[[\text{at least } (A)(p)(w)]\] = \(\text{Exh}(A)(p)(\text{the}'(\text{est}'(E_w)(H^{\text{less}}_{A})))\)

\(H^{\text{less}}_A(w)(d) = d\text{-few propositions in } A \text{ are true in } w.\)

\textbf{Why is the implicature in question not present?}

\textbf{Intuition:}
Presuppositions of \textit{at least}/\textit{at most}: \(E_w\) must contain two worlds in which the number of true propositions in A is different. Hence the content of a sentence in which \textit{at least} has matrix scope can’t determine how many proposition in A are true.

\textbf{Implementation:}

(30) John is at least 6 feet tall

(31) \(\text{Exh}(A)(\text{at least}(A') \text{ John is } 6F_T \text{ tall})(w) = 1 \text{ iff}
\[\text{at least}(A') \text{ John is } 6F_T \text{ tall } (w) = 1 \text{ and}
\forall d \neq 6 \ (\text{at least}(A') \text{ John is } dF_T \text{ tall } (w) = 0) \text{ iff}
\[\text{at least}(A') \text{ John is } 6F_T \text{ tall } (w) = 1\)
4.2. Possible Advantage


(32) a. John has been in Paris since July. (both U and E readings)
    b. John has been in Paris at least since July. (only U reading)

If the UDM is correct, as well as the lexical entries above, we predict that at least and at most just like only will be inconsistent with N-open properties:

(33) a. I am at least sure that he’s been in Paris since JULY.
(34) a. John has been in Paris at most since July. (only U reading)
    b. I can at most guarantee that he’s been in Paris since July.

(35) a. * John at least has not smoked 30_{F} cigarettes.
    b. * John at most has not smoked 30_{F} cigarettes.

Benjamin Spector (p.c.)

(36) a. John is at least not allowed to smoke 30_{F} cigarettes.
    b. John is at most not allowed to smoke 30_{F} cigarettes.

5. A different approach to (p or q) or both

Hurford’s Generalization: A or B is infelicitous when B entails A.²

(37) a. ??John is an American or a Californian.
    b. ??I was born in France or Paris.

Hurford used this generalization to argue for a strong meaning for disjunction (ExOR):

(38) I will apply to Cornell or UMASS or to both.

We can extend this argument to other scalar items:

(39) a. I will read two books or three.
    b. I will do some of the homework or all of it.

(38') Exh(C)(I will apply to Cornell or F UMASS) or [I will apply to Cornell and UMASS]

What happens if we add yet another exhaustive operator? Nothing.
What happens if we apply Sauerland’s algorithm? No secondary implicatures.

6. Remaining Implicatures:

(40) Who came to the party?
   2 or more boys.
   Implicature: Girls didn’t come.
   Only 2 or more boys. (entails: girls didn’t come)

(41) Only(A)(2 or more boys)(w)???

(42) Who came to the party?
   2 boys.
   Only 2 boys.
   Implicature: exactly 2 boys came and Girls didn’t come.

Spector (2004):
A = closure under \{\land, \lor\} of the Hamblin denotation of the question.

(9) S&S-based:
only(A<st,t>)(pst)(w) ⇔ p(w) & ∀q∈A
   \[K¬q \text{ is consistent with } Kp ∧ PI(A)(p)] → q(w) = 0

   PI(A)(p) = \cap \{¬Kq: q∈A \text{ and } q⇒p\}

Assume the following domain: b1, b2, b3 g1,g2

(43) Only(A)(2 boys came to the party)(w) ⇔
   2 boys came to the party in w and
   not (((b1 & b2) or (b1 & b3) or (b2 & b3)) and g1) and
   not (((b1 & b2) or (b1 & b3) or (b2 & b3)) and g2) and
   not (b1 & b2 & b3)

(44) Who came to the party?
   At least 2 boys.
   Implicature ??? Girls didn’t come.
   *Only at least 2 boys

If there is an implicature, we might consider the following:

(9') only(A<st,t>)(pst)(p)(w) ⇔ p(w) & ∀q∈A
   \[K¬q \text{ is consistent with } Kp ∧ PI(A)(p) ∧ \text{Presuppositions(p)}] → q(w) = 0