The Universal Density of Measurement

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1. Goal: To argue that degree/measurement scales are always dense:

(1) The Universal Density of Measurements (UDM): Measurement Scales that are needed for Natural Language Semantics are *always* dense.

In other words, we will argue for two claims:

(2) a. The Intuitive Claim: Scales of height, size, speed, and the like are dense.
b. The Radical Claim: All Scales are dense; cardinality in not a concept of NLS.

Precursor:

Von Stechow, (1984):

(3) *John is taller than Bill is* is true iff the *maximal* degree of height that Bill possesses is below John's height.

Pinkal (1989; Via Heim): This will not allow us to account for the meaning of:

(4) John is taller than he has ever been before.

Pinkal's argument is based on the assumption that the domain of degrees of height (and the domain of times) is dense. Von Stechow might respond to this challenge by denying the density assumption, e.g. by postulating "minimal granularity".

What is to follow is a detailed argument against this response.

Potential consequences of the Radical Claim:

There is a possible connection with work in experimental psychology which aims to discover the origins of human reasoning about quantities and numerosities. (Carey, Spelke, Gelman and Gallistel, among many others.)

Recurring hypothesis: Prior to the development of adult arithmetic there is a core system that allows the measurement (or at least the estimation) of quantities, but crucially does not have access to anything like the notion of a natural number.

1. Implicatures and *only*

1.1. The Puzzle:

Krifka (1998):

(5) a. John has 3 children.
Implicature: John doesn't have 4 children.
b. John has more than 3 children.
*Implicature: John doesn't have more than 4 children.

A possible response (somewhat vague): (5)b doesn't have the implicature that John has exactly 4 children because there is a simpler (briefer) way to convey that information (Maxim of brevity).

But, assume it is presupposed that Bill has exactly 3 children:

(6) a. John has more children than Bill does.
*Implicature: John has exactly four children.
b. John has two more children than Bill does.
Implicature: John has exactly 5 children.

1.2. An Identical Puzzle with *only*:

(7) John has very few children.
a. He only has 3_F children.
b. *He only has more than 3_F children.[1]

Fox (2004):
(8) The *only* implicature generalization (OIG): Utterance of a sentence, *S*, as a default, licenses the inference/implicature that (the speaker believes) *onlyS'*, where *S'* is (a minimal modification of) *S* with focus on scalar items.

1.3. The Intuitive Claim

(9)a. John weighs 120 pounds.
Implicature: There is no degree, d, greater than 120, s.t. John weighs d pounds.
b. John weighs more than 120 pounds.
*Implicature: There is no degree, d, greater than 120, s.t. John weighs more than d pounds.
c. *John only weighs more than 120_F pounds.

In this case, the explanation of the facts is quite transparent:

[1] All of the effects discussed in this paper do not hold when the context specifies a discrete set of relevant alternatives. We will get back to that.

(9)c presupposes that John weighs more than 120 pounds, 120 + ε pounds; John, therefore weighs more than 120 + $\varepsilon/2$ pounds, and, hence, there is a degree, *d*, greater than 120 such that John weighs more than *d* pounds.

If The *Intuitive* Claim, (2)a, is true, the unacceptability of (9)c follows, since 120 + $\varepsilon/2$ has to be a member of the set relevant domain of degrees.

(9)b would follow if implicatures are represented by a covert operator a kin in meaning to *only exh*. (As suggested by Krifka (1995); see also Groenendijk and Stokhof 1984 and van Rooy (2002)).[2]

$\text{Exh}(C)(\varphi)(w) = 1 \text{ iff } \forall\Psi\in C, \Psi(w)=1, \varphi \text{ entails } \Psi$

1.4. The Radical Claim

We can provide exactly the same explanation for the (5)b and (7)b and if *all* degree domains are dense.

(5)b John has more than 3 children.
*Implicature: John doesn't have more than 4 children.

(7)b *John only has more than 3_F children.

(7)b asserts that for any degree *d* greater than 3, John doesn't have more than *d* children.

Without the UDM, there would be no problem. The set of degrees relevant for evaluation would be, as is standardly assumed, possible cardinalities of children (i.e., 1, 2, 3,…). The sentence would then assert that John doesn't have more than 4 children. Since it presupposes that John has more than 3 children, it would end up conveying that he has exactly 4.

If The Radical Claim is assumed, however, we are in exactly the same situation that were in when discussing (9)c.

The presupposition of course remains the presupposition that John has more than 3 children. However, the assertion would now not just exclude 4 as a degree exceeded by the number of John's children. It would also exclude any degree between 3 and 4.

The lack of an implicature in (5)b will follow in exactly the same way, if *exh* is needed to generate implicatures.

[2] It will also follow from the neo-Grician framework, if Horn Scales are assumed to be dense. But, we will assume exh, because otherwise we will not be able to derive the desired consequences from The Radical Claim. (See Fox 2004.)

1.5. Further Evidence

Universal Modals should circumvent the effect:

(10) a. I can only say with certainty that John weighs more than 120_F pounds.
b. I can only say with certainty that John has more than 3_F children.

(11) a. I was only able to demonstrate that this refrigerator weighs more than 120_F pounds.
b. I was only able to demonstrate that this candidate received more than 500_F votes.

(12) a. I can say with certainty that John weighs more than 120 pounds.
Implicature: I can only say with certainty that John weighs more than 120_F pounds.
b. I can say with certainty that John has more than 3 children.
Implicature: I can only say with certainty that John has more than 3_F children.

(13) a. I was able to demonstrate that this refrigerator weighs more than 120 pounds.
Implicature: I was only able to demonstrate that this refrigerator weighs more than 120_F pounds.
b. I was able to demonstrate that this candidate received more than 500 votes.
Implicature: I was only able to demonstrate that this candidate received more than 500_F votes.

One can presuppose it to have been demonstrated that x weighs more than 120 pounds, and subsequently assert consistently that there is no degree d, greater than 120, such that it has been demonstrated that x weighs more than d pounds. The reason for this is obvious: a demonstration that x weighs more than d pounds doesn't entail a demonstration that x weighs d + ε pounds for some specific degree ε.

It is of possible to make sense of this fact in possible world semantics, assuming a dense set of world corresponding to the degrees: for every ε there could be a world consistent with the demonstration such that in that world John weighs less than d + ε pounds.

Existential Modals shouldn't circumvent the effect:

(14) a. You're required to read more than 30 books.
Implicature: There is no degree greater than 30, d, s.t. you are required to read more than d books.
b. You're only required to read more than 30_F books.

(15) a. You're allowed to smoke more than 30 cigarettes.
*Implicature: There is no degree greater than 30, d, s.t. you are allowed to smoke more than d cigarettes.
b. *You're only allowed to smoke more than 30_F cigarettes.

If you are allowed to smoke more than 30 cigarettes, it follows that you're allowed to smoke 30 + ε cigarettes, for some degree ε. This consequence would contradict the potential implicature or the sentence with *only*.

A similar consequence does not follow when you are required to read more than 30 books. If you are required to read more than 30 books, there need not be a degree ε, such that you are required to read 30 + ε books, and incoherence is avoided in exactly the manner we've discussed when we accounted for (10)-(12).

In possible world:

Existential Modals don't obviate the problem:

-If you are allowed to smoke more than 30 cigarettes, we say that there is a world, *w*, (compatible with your requirements) and there is a degree ε, such that you smoke 30 + ε cigarettes in *w*.
-By the UDM: In *w*, there is a degree greater than 30 (30 + $\varepsilon/2$) such that you smoke more than that degree of cigarettes.
-By the commutativity of two existential quantifiers, there is a degree greater than 30 such that you are allowed to smoke more than d cigarettes.

Universal Modals obviate the problem (as we've seen):

-If you are required to read more than 30 books, we say that for every world, *w*, (compatible with your requirements) there is a degree ε, such that you read 30 + ε books in *w*.
-By the UDM: for every such *w*, there is a degree greater than 30 (30 + $\varepsilon/2$) such that you smoke more than that degree of cigarettes.
-But a universal and an existential cannot commute freely, hence we cannot conclude that there is a degree greater than 30 such that you are required to read more than d cigarettes.

2. Negative Islands: Definite Description and Questions[3]

2.1 Rullmann (1995)

(16) John didn't read many of these books?
Question: Which books did John not read?

(17) John doesn't weigh 190 pounds.
Question: *How much does John not weigh?

Rullmann's proposal (following van Stechow 1984):

(18) How much does John weigh?

(19) How much/many φ?
What is the *maximal* degree d st. φ(d)?

[3] This section is concerned with degree constructions. Whether or not the treatment can be extended to other negative islands is left open.

(18)' How much does John weigh?
What is the maximal degree d such that John weighs (at least) d pounds?

There can be no maximal degree that would answer the question in (17).

(17)' *How much does John not weigh?
What is the maximal degree d such that John does not weighs (at least) d pounds?

2.2 Evidence in favor of Rullmann

(20) a. *I have the amount of water that you don't.
cf. I have the amount of water that you do.
b. I have an amount of water that you don't.

The unacceptability of (20)a can receive the same explanation that Rullmann gives for the unacceptability of the question in (17). Definite descriptions presuppose that the predicate they combine with has a maximal element in its denotation (Link 2003):

(21) *The* φ is defined only if there is a maximal object x st. $\varphi(x)$.
When defined *the* φ refers to the maximal object x st. $\varphi(x)$.

Rullmann assumes, in effect, that degree questions request the value of a definite description: What is *the* degree d s.t. John weighs d pounds? (cf. Spector in progress and 2.4. below)

2.3 Evidence against Rullmann (Beck and Rullmann 1999)

(22) How much flour is sufficient to bake a cake?

If Maximality is to play a role in the semantics of questions, what is maximized is informational content:

(23) How much/many φ?
a. Rullmann (1995):
- What is the maximal degree d st. $\varphi(d)$?

b. modification based on Beck and Rullmann (1999):
- What is the degree d that yields the most informative among the true propositions of the form $\varphi(d)$? (a special case of Dayal 1996)

2.4 Definite Descriptions are the same (Fintel, Fox, and Iatridou, in progress)

(24) I have the amount of flour sufficient to bake a cake.

Under the standard semantics the definite description in (24) will always yield a presupposition failure since there could never be a maximal amount of flour sufficient to bake a cake. In order to deal with this observation,

(25) *The* φ is defined only if there is a unique individual x such φ(x) is a maximally informative proposition among the true propositions of the form φ(x).
When defined *the* φ refers to the individual x st. φ(x) is the maximally informative true proposition of the form φ(x). [4]

$[\![\text{the}]\!] = \lambda P_{<e,st>}\lambda w{:}\exists x P(x)(w)=1 \text{ and } \forall y\neq x\ (P(y)(w)=1 \rightarrow P(x) \text{ asymmetrically entails } P(y)).$
$(\iota)xP(x)(w)=1 \text{ and } \forall y\neq x\ (P(y)(w)=1 \rightarrow P(x) \text{ asymmetrically entails } P(y)).$

This semantics just like Rullmann's, (23)b, has the advantage of providing a unified account for the variation between a maximality and a minimality effect.

Furthermore, both render Rullmann's original explanation of the negative island effects in definites and questions [(17), (20)a] unavailable.

A relevant aside (further evidence for the new lexical entry):

(26) The Greek soldiers who together can destroy the Trojan army
*The **minimal** set of soldiers who together can destroy the Trojan army*

(27) The questions such that if you answer all of them correctly, you pass the test
*The **minimal** set of questions such that if you answer all of them correctly, you pass the test*

Suppose that I am trying to fit books into a book shelf and there are 5 books (x, y, z, w, v) of various sizes and also shelves of various size (*a, b, c*...). Suppose that book x together with book y fit perfectly into shelf *a*, and book x, y, and z together fit perfectly into shelf *b*. But no other combination of books fits perfectly into a shelf.

(28) #Pass me the books that together fit perfectly into a shelf.
*Undefined in the context above, because there is no **minimal** set of books that together fit perfectly into a shelf.*

[4] The required lexical entry is the following:
$[\![\text{the}]\!] = \lambda P_{<e,st>}\lambda w{:}\exists x P(x)(w)=1 \text{ and } \forall y\neq x\ (P(y)(w)=1 \rightarrow P(x) \text{ asymmetrically entails } P(y)).$
$(\iota)xP(x)(w)=1 \text{ and } \forall y\neq x\ (P(y)(w)=1 \rightarrow P(x) \text{ asymmetrically entails } P(y)).$

2.5. Negative Islands and The intuitive part of the UDM

(29) *How much does John not weigh?

(29) should ask for **the maximally informative** proposition among the true propositions of the form John does not weight d-pounds.

The most informative proposition of this form is the proposition that John does not weigh d pounds where d is the minimal degree d for which this proposition is true.

However, given The Intuitive Claim, (2)a, there is no minimal degree that yields a true proposition of the relevant form since the relevant set of degrees is dense.

Suppose that John weighs exactly 150 pounds. This means that for any degree d in the set of degrees greater than 150, John doesn't weigh d pounds. From density it follows that there is no minimal member in this set.

Exactly the same reasoning accounts for the unacceptability of (20)a. However there are further predictions.

[+NI]:= An environment that involves movement of a degree operator across negation
[+MV]:= An environment in which Max_{inf} is not defined

(30) a. Prediction 1: [+NI,-MV] cases should be acceptable.
b. Prediction 2: [-NI,+MV] cases should be unacceptable.

Prediction 1:

(31) a. How much are you sure that this vessel won't weigh?
b. How much radiation are we not allowed to expose our workers to?
c. The amount of radiation that we are not allowed to expose our workers to is greater than we had thought.
d. The amount of money that you are sure that this stock will never sell for is quite high. (Are you sure that your estimation is correct.)

In all four examples it is the *minimal* degree that is relevant.

Under our density assumption, these are all [-MV] cases and their acceptability is therefore predicted.

To see this, focus on (31)a. Even if the domain of degrees is dense, as we are assuming, there could be a minimal degree d such that the addressee is sure that the vessel won't weigh d pounds. Even though there can be no minimal degree d that the vessel doesn't weigh d pounds, there could be a minimal upper bound to possible weights.

We take it to be necessary that there is a degree d such that the vessel weighs exactly d pounds. In other words, the set of degrees d such that the vessel weighs at least d pounds is necessarily a closed interval.

Consequently, given density, the complement set is necessarily an open interval which cannot have a minimal member.

However, the set of degrees d such that there is *certainty* (on someone's part) that the vessel doesn't weigh at least d could be a closed interval with a minimal member.

To see this, consider the fact that this set could also be described as the set of degrees d for which there is no possibility that the vessel weighs d pounds. This set is the complement set of the set of degrees for which there is a possibility that the vessel weighs d pounds and this set in turn could be an open interval.

For example, it will be an open interval if there is a degree d such that the vessel can't weigh d pounds, but for any smaller degree d', there is a world (in the modal base) in which the vessel weighs exactly d' pounds.

Universal and Existential modals are different:

(32) a. How much money are we not allowed to bring in to this country?
b. *How much money are we not required to bring in to this country?

(33) a. How much money are we requited not to bring in to this country?
b. * How much money are we allowed not to bring in to this country?

(34) a. How much radiation is the company not allowed to expose its workers to?
b. *How much food is the company not required to give its workers?[5]

(35) a. How much radiation is the company required not to expose its workers to?
b. *How much food is the company allowed not to give its workers?

Suppose that there is an answer to (34)b (equivalent to (35)b). Let's say the answer is *3 bowls of rice*. If there is an allowed world, w_0, in which the company doesn't give its workers 3 bowls of rice, there is an ε such that the company gives it's workers exactly 3-ε bowls of rice ($0<\varepsilon \leq 3$). But given density there is a more informative answer. In w_0 the company doesn't give its workers 3-$\varepsilon/2$ bowls of rice, and the proposition that this is allowed is more informative. We see the same contrast with definite description and the account is of course the same.

[5] Ignore the following irrelevant reading: What is the amount of food such that there is food in that amount and the company is not required to give that food to its workers. To avoid this problem:
(i) (When you enter the country) How much money are you not allowed to have.
(ii) *(When you enter the country) How much money are you not required to have.

(36) a. The amount of radiation that the company is not allowed to expose its workers to is very low.
b. *The amount of food that the company not required to give its workers is very high.

Prediction 2

(37) a. *Before when did John arrive? (Fintel and Iatridou 2002)
b. Before when do you have to arrive?

(38) a. *I arrived at the time before noon.
b. ?I arrived at a time before noon.

2.6. On the universality of the UDM; Evidence for The Radical Claim

(39) *How many kids do you not have?

(40) a. If you live in China, how many children are you not allowed to have?
b. How many days a week are you not allowed to work (according to the union regulations)?
c. How many soldiers is it (absolutely) certain that the enemy doesn't have?

(41) a. ???Combien John n'a-t-il pas lu de livres?
How many John n'has-he not read of books
b. ? Combien peux-tu me dire avec (absolue) certitude que John n'a pas lu de livres?
How many can-you me tell with (absolute) certainty that John has not read of books
(Benjamin Spector, pc)

(42) a. *Combien Jean n'a-t-il (pas) d'enfants?
How many John n'has-he not of children
b.? Combien les chinois ne peuvent ils (pas) avoir d'enfants?
How many the chineese n'alloweed-them not have of-children?
(Valentine Hacquard, pc)

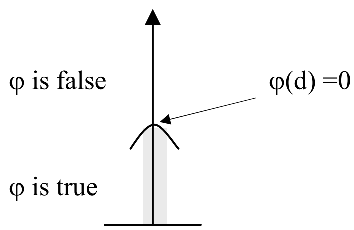
(43) a. *If you live in Sweden, how many children are you not required to have?
b. *How many days a week are you not required to work (even according to the company's regulations)?
c. *How many soldiers is it possible that the enemy doesn't have?

3. Exhaustivity and Density: a unified account

3.1. The results of Section 1

Let φ be an upward monotone property of degrees (of type <d,st>) which necessarily describes an open interval.[6]

In every world:



It will necessarily be false to say that there is some degree d such that it is the maximal degree such that φ is true of that degree in w.

Equivalently (given upward monotonicity), it will be false to say that there is a degree d such that φ(d)(w) is true and φ(d) is more informative than φ(d') for every d' such that φ(d')(w) is true.

The basic result of section 1:

1. A simple property of degrees based on comparatives, e.g. *λd. John has more than d children*, is upward monotone.
2. Assuming the UDM it also necessarily describes an open interval.
3. Hence, it follows that a statement that there is a maximal (or maximally informative) degree that satisfies it would be infelicitous (whether it results from *only* or a covert exhaustive operator).

Universal modal operators can close an interval:

If we append a universal modal operator, the modified property, λd. $\Box$φ(d), no longer necessarily describes an open interval. The modal could quantify over a dense set of worlds that corresponds to the set of degrees with the result that the modified property describes closed interval.

[6] φ is upward monotone if for every d_1,d_2: $d_1>d_2$ iff $φ(d_1)$ is more informative than $φ(d_2)$ (asymmetrically entails it).

φ necessarily describes an open interval if for every world w, λd.φ(d)(w) denotes an open interval. (There is a degree d such that φ(d)(w)=0 but for every degree d' smaller than d φ(d')(w)=1.)[6]

Hence, complex properties of degrees such as 'λd. you are required to read more than d books' can be the source of an implicature or a parallel sentence with *only*.

Proof:

Let,

I_k := a series of open interval converging to [0, *a*] from the top, e.g. $[0, a + 1/k)$

w_k a series of worlds, s.t. for every k, $\lambda d.\varphi(d)(w_n) = I_k$.

Modal Base for $\Box$ in w_0 := the members of the series w_k

It is easy to see that:

$\lambda d.[\Box\varphi(d)](w_0) = [0, a]$.

Let $d \notin I$. I.e., $d > a$.
Given convergence from the top: $\exists N$, for all n bigger than N, $\varphi(d)(w_n)=0$
Hence, $\Box\varphi(d)=0$.

Let $d \in I$. Given that I is a subset of every I_n, $\Box\varphi(d)=1$.

Existential modal operators cannot close an interval:

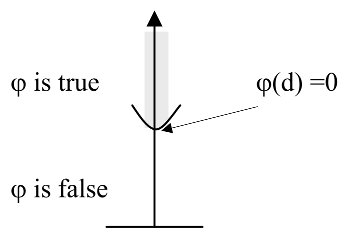
Appending an existential modal operator is of no help: the property $\lambda d.\ \Diamond\varphi(d)$ necessarily describes an open interval.

Proof:
Assume that $\lambda d.\ \Diamond\varphi(d)$ denotes a closed interval (in a world w)
Let, $d' = \text{Max}\ (\lambda d.\ [\Diamond\varphi(d)](w))$
Hence, $[\Diamond\varphi(d')](w)=1$.
Hence, $\exists w'$ in the modal base such that $\varphi(d')(w')=1$.
But since φ necessarily describes an open interval, $\exists d'' > d'\ [\varphi(d'')(w')=1]$.
Hence, $[\Diamond\varphi(d'')](w) =1$ contrary to assumption.

This explains the fact that complex properties of degrees such as 'λd. you are allowed to smoke more than d cigarettes' cannot support an implicature and are unacceptable with *only*.

3.1. The results of Section 2

Let φ be a downward monotone property of degrees (of type <d,st>), which necessarily describes an open interval. For every world:



It will necessarily be false to say that there is some degree d such that it is the minimal degree such that φ is true of that degree in w.

Equivalently (given downward monotonicity), it will be false to say that there is a degree d such that φ(d)(w) is true and φ(d) is more informative than φ(d') for every d' such that φ(d')(w) is true.

The basic result of section 2:

1. A simple property of degrees based on negation, e.g. *λd. John doesn't have d children*, is downward monotone.
2. Assuming the UDM it also necessarily describes an open interval.
3. Hence, it follows that a statement that there is a minimal (or maximally informative) degree that satisfies it would be infelicitous (whether it results from a degree question or a definite article).

Universal modal operators can close an interval:

If we append a universal modal operator, the modified property, λd. $\Box$φ(d), no longer necessarily describes an open interval. The modal could quantify over a dense set of worlds that corresponds to the set of degrees with the result that the modified property describes closed interval.

Hence, complex properties of degrees such as 'λd. you are required not to have d children', which is equivalent to 'λd. you are not allowed to have d children' can serve as arguments of a definite article or a question operator.

Proof:

Let,

$I_k := (a - 1/k, \infty) \rightarrow [a, \infty)$ (from the bottom)

w_k a series of worlds, s.t. for every k, $\lambda d.\varphi(d)(w_n) = I_{k.}$

Modal Base for $\Box$ in w_0 := the members of the series w_k

It is easy to see that:

$\lambda d.[\Box\varphi(d)](w_0) = [a, \infty)$.
Let $d \notin I$. I.e., $d < a$.
Given convergence from the bottom: $\exists N$, for all n bigger than N, $\varphi(d)(w_n)=0$
Hence, $\Box\varphi(d)=0$.

Let $d \in I$. Given that I is a subset of every I_n, $\Box\varphi(d)=1$.

Existential modal operators cannot close an interval:

Appending an existential modal operator is of no help: the property $\lambda d.\ \Diamond\varphi(d)$ necessarily describes an open interval.

Proof:
Assume that $\lambda d.\ \Diamond\varphi(d)$ denotes a closed interval (in a world w)
Let, $d' = \text{Min}\ (\lambda d.\ [\Diamond\varphi(d)](w))$
Hence, $[\Diamond\varphi(d')](w)=1$.
Hence, $\exists w'$ in the modal base such that $\varphi(d')(w')=1$.
But since φ necessarily describes an open interval, $\exists d'' < d'\ [\varphi(d'')(w')=1]$.
Hence, $[\Diamond\varphi(d'')](w) =1$ contrary to assumption.

This explains the fact that complex properties of degrees such as 'λd. you are allowed not to have d children' (λd. you are not required to have d children') cannot serve as arguments of a definite article or a question operator.

3.3. The Generalization

Let φ be a d monotone property of degrees (of type <d,st>), which necessarily describes an open interval (on the informative edge)

(44) Constraint on Interval Maximization (CIM): N-open monotone properties cannot be maximized by $MAX_{inf.}$

$MAX_{inf}(\varphi_{<\alpha,st>})(w)$ = the $x \in D_\alpha$, s.t., $\varphi(x)(w) =1$ and $\forall\Psi(\Psi(x)(w)=1 \rightarrow \varphi$ entails $\Psi)$.

The fact that the CIM plays an explanatory role in accounting for the status of expressions in natural language can be taken as evidence that natural language has N-open properties, i.e. for The Intuitive Claim, (2)a.

The fact that the CIM seems to be at work in *all* degree constructions (even those that putatively make reference to cardinality) constitutes our argument for the universality of the UDM, i.e. for The Radical Claim, (2)b.

Additional predictions:

(45) Basic Consequence:
If φ is an N-open monotone property of degrees, then the following should be unacceptable
a. *Exh$\varphi(d_F)$
b. *Only$\varphi(d_F)$
c. *d? $\varphi(d)$
d. *the $\varphi(d)$

(46) Consequence for universal modals:
A universal modal can close an interval, hence even if φ is an N-open monotone property of degrees, the following should be acceptable
a. Exh$\Box\varphi(d_F)$
b. Only$\Box\varphi(d_F)$
c. d? $\Box$ $\varphi(d)$
d. the $\Box\varphi(d)$

(47) Consequence for existential modals:
An existential modal cannot close an interval. Hence, if φ is an N-open monotone property of degrees, then the following should be unacceptable
a. *Exh$\Diamond\varphi(d_F)$
b. *Only$\Diamond\varphi(d_F)$
c. *d? $\Diamond$ $\varphi(d)$
d. *the $\Diamond\varphi(d)$

In section 1 we saw evidence for the UDM under the assumption that a and b hold for upward monotone degree properties. Can we find evidence based on c and d?

In section 2, we saw parallel evidence under the assumption that c and d hold for downward monotone properties. Can we find evidence based on a and b?

3.4. Implicatures, *Only*, and Negation

It is well known that scalar implicatures are predicted to "reverse" in downward entailing environments. (See, in particular, Chierchia 2002.):

(48) a. John did some of the homework.
Implicature: John didn't do all of the homework
b. John talked to Bill or Sue.
Implicature: John didn't talk to both Bill and Sue.

(49) a. John didn't do all of the homework.
Implicature: John did some of the homework.

b. John didn't talk to both Bill and Sue.
Implicature: John talked to one of the two.

The reversal is of course expected if implicatures are generated by a covert exhaustive operator (exh) which makes use of MAX_{inf}.

From this perspective it is somewhat surprising that the sentences in (50)a-b do not give rise to parallel implicatures.[7]

(50) a. John didn't smoke 30 cigarettes.
*Implicature: John smoked 29 cigarettes.
b. John didn't read 30 books.
*Implicature: John read 29 books.

The UDM, however, provides an account.

For the sentences in (50) to have an implicature, *exh* would have to be employed. The resulting interpretation would be equivalent to the conjunction of the standard semantics of the prejacent and the assertion that all stronger alternatives are false. For example (50)a would make the assertion in (51).

(51) John didn't smoke 30 cigarettes and for all degrees d smaller than 30 John smoked d cigarettes.

(51) is of course contradictory if the UDM is assumed. As is by now familiar, if John didn't smoke 30 cigarettes, then the exact degree, d, of cigarettes that he smoked is below 30. By the UDM there has to be a degree d' between 30 and d. Since John didn't smoke d' cigarettes (51) cannot be true.

Under the UDM, this is just an instantiation of (45)a for downward monotone properties. The UDM also make a prediction based on (46)a, namely that the problem with adding the exhaustive operator should disappear with the introduction of a universal modal operator:

(52) John is not allowed to smoke 30 cigarettes.
Implicature: John is allowed to smoke 29 cigarettes.

(53) John is required not to smoke 30 cigarettes.
Implicature: John is allowed to smoke 29 cigarettes.

Finally, we make the prediction based on (47) that an existential modal will not obviate the CIM:

(54) John is not required to read 30 books.
*Implicature: John is required to read 29 books.

---

[7] The observations in this section grew out of attempts to grapple with observations made by B. Spector (in progress).

For some reason that is not completely clear to us, the facts seem to change when a non-round numeral is used. We don't understand this phenomena, but hope that it can be made consistent with our proposal once the pragmatics considerations that enter into choosing a level of precision is taken into account. See, e.g. Krifka 2004.

(55) John is allowed not to read 30 books.
*Implicature: John is required to read 29 books.

Parallel facts are predicted for *only* (instantiating the b cases in (45)-(47)). The predictions seem to go in the right direction.[8]

In (56) we see that *only* can associate with scalar items across negation. These examples are not perfect, but contrast markedly with the sentences in (57), which violate the CIM under the UDM. This contrast corroborates the predictions of the UDM based on (45)b.

(56) a. John only has not done all_F of the homework.
b. John only has not spoken to $both_F$ Bill and Sue.

(57) a. *John only has not smoked 30_F cigarettes.
b. *John only has not read 30_F books.
c. *John only does not weigh 190_F pounds.

The addition of a universal modal, (59), and the move to the equivalent construction with the existential modal, (58), result in sentences that are quite good, by contrast.[9]

(58) a. John is only required not to smoke 30_F cigarettes.
b. John is only required not to weigh 190_F pounds.

(59) a. John only has not been allowed to smoke 30_F cigarettes.
b. John only has not been allowed to weigh 190_F pounds.

Finally, the addition of an existential modal, (60), and the move to the equivalent dual, (61), are of no help.

(60) *John is only allowed not to read 30_F books.

(61) *John only has not been required to read 30_F books.

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[8] Not all the data is as clean as we would like it to be. We think that this might be due to an independent syntactic constraint on the positioning of only. This constraint, we think, disallows only from appearing immediately to the left of negation. The way we circumvent this is by placing an auxiliary between negation and *only*, which might not be perfect to all ears. See, however, Spector (2004) for an argument that this constraint might capture additional facts that we derive from the UDM.

As we've seen the exhaustivity operator cannot be restricted by the same constraints. This is a counter-example to the OIG as it is stated, but not to the account that it motivates, which appeals to an exhaustivity operator. The two operators (although similar) need share all syntactic properties.

[9] Though see footnote 8, re (58).

3.5. Questions, Definites, and Comparatives

We now want to check the predictions of the UDM for upward monotone properties given (45) (c) and (d), specifically, the prediction that properties that are N-open under the UDM cannot be maximized by a question operator or a definite article.

Furthermore we want to check that the effect will be obviated by the universal modal operator, (46), but not by the existential modal operator, (47).

Our test for this prediction is unfortunately far from perfect, given independent constraints on the formation of the relevant questions and definite descriptions. Nevertheless, the facts seem to us to go in the right direction.

(62) a. *More than how many books did John read?
b. ??More than how many books does John have to read?
b. ??/*More than how many cigarettes is John allowed to smoke?

(63) a. *How many feet is John under?
b. ?How many feet do you have to be under to take this ride?
(Steve Yablo, pc)

(64) a. *(I tried to eat) the amount of food such that you ate more than that.
b. ?(I tried to eat) the amount of food such that you are required to eat more than that.
c. *(I tried to eat) the amount of food such that you are allowed to eat more than that.

It is worthwhile investigating whether there are languages in which the interfering constraints are not at work, for example, are there languages that freely allow degree resumption? If there are, the contrasts ought to be sharper.

3.6. *Allowed* and Free choice

(65) a. You are allowed to arrive before 10.
Implicature: You are not allowed to arrive before 9.
b. Before when are you allowed to arrive?

4. Problems and Consequences

4.1. Modularity

A killer problem:

(66) a. I can say with certainty that John has more than 3 children.
Implicature: I can only say with certainty that John has more than 3_F children.
b. I can only say with certainty that John has more than 3 children.

When you think of the truth conditions of these sentences, it seems that only the integers are in the domain of quantification. Same problem for questions.

(67) How many kids can you say with certainty that doesn't have?

There is a more basic problem (with respect to which we were hoping that you would suspend disbelief): the rounding/granularity problem.

(68) John is six feet tall

Sentences are evaluated in a context which specifies (or is assumed to specify) a level of granularity, G. We can think of G as an equivalence relation.

(69) $[[\text{John is six feet tall}]]^{G,w} = 1$ iff John's height, d, is such that G(d)(six-feet).

Problem:

(70) $[[\text{Only[A][John has more than } 3_F \text{ Children]}]]^{C/G,w} = 1$ iff for every proposition in A, φ, if φ(G)(w)=1, then the proposition that John has more than 3 children is more informative than φ under C/G.

Ψ is more informative than φ under C/G iff every world w (in C) that satisfies Ψ under G also satisfies φ under G.

If the "under G" part is added to the definition of informativity, we loose our account of unacceptability.

We are therefore forced to conclude that G doesn't enter the picture at the level at which the CIM is evaluated, though it does enter when the truth of a sentence is evaluated in a particular context.

This is more or less the divide between syntax/semantic and pragmatics. The CIM is evaluated in syntax/semantics rather than pragmatics.

We thus postulate a deductive system, DS, (Fox 2000, Gajewski 2002/2003) in which sentences are evaluated and ruled out if they are can be proven to be unusable (contradictions, tautologies, etc.). See Gajewski's for a detailed discussion of what this system might look like.

In DS sentences are ruled out if they can be proven to be contradictory (under the stringent granularity, =).

Once a sentence passes DS, it is evaluated in a particular context, where a level of granularity may affect the interpretation.

4.2. Syntactic Contextual Restriction

Kroch (1989): When the context provides an explicit set of alternatives, negative islands are circumvented:

(71) Among the following, please tell me how many points Iverson didn't score?
a. 20 b. 30 c. 40 d. 50

What is the most informative degree in C, s.t. Iverson didn't score d points?
C = {20, 30, 40, 50}

This extends to implicatures and *only*.
(72) Iverson sometimes scores more than 30 points. But today he only scored more than 20_F.

(73) A: How many points did Iverson score last night?
B: I don't know.
A: Was it more than 10, more than 20 or more than 30.
B: He scored more than 20 points
Implicature: he didn't score more than 20.

Exh/Only[C] [Iverson scored more than 20 points]
{that Iverson scored more than 30 points, that I. scored more than 30 points}

4.3. Degrees and the Mass Count distinction

Problem: If cardinality doesn't exist in NLS, how do we make sense of the mass count distinction?

More specifically, it is standardly assumed that plural count nouns are properties of a domain of individuals that have cardinality. (e.g. closure under + based on the domain of atoms).

So what would it mean to say that John 3.5 kids?

This problem needs to be addressed on independent grounds (Krifka)

(74) a. I read 3.5 books.
b. This weighs 10.5 pounds.

See Magri (2004) for various arguments that we should think differently about plurality.

4.4. Do degree questions make use of $\mathbf{Max_{inf}}$?

Beck and Rullmann:

(75) A: How many people can play soccer?
What is the number d such that it is possible (given the laws of soccer) for exactly d people to play together?
B: 6 people (indoor soccer), 8 people (small field) and 11 people (regular)

(76) A: How many courses are you allowed to take?
What is the number d such that it is possible (given the laws of the school) for you to take exactly d classes?
B: Any number between 4 and 6.

Irene's Objection:

(77) A: How much money are you not allowed to bring into this country?
B: \$10,000
B': The maximum allowed is \$10,000.
= You're not allowed to bring in any amount that exceeds \$10,000.

Tentative Response:

Degree questions don't make direct reference to Max_{inf}:

$[\![\text{how many } \varphi]\!] = \{p: \exists d\ (p = \varphi(d)\}$.

A proper answer to a degree question is the following (Heim's 1994, Answer-1):

$Answer(Q)(w) = \lambda w'(\forall p \in Q\ [p(w)=1 \rightarrow p(w')=1])$ (the conjunction of all true answers).

However: There is indirect reference to Max_{inf}:

Proposed Constraint: A question Q is only felicitous if it's possible for the proper answer to Q to be a member of Q [$\Diamond\lambda w(Answer(Q)(w) \in Q)$].

$Answer(Q)(w) \in Q$ iff $Max_{inf}(\varphi)(w)$ is defined (where φ is the question property).

If this constraint holds, a Question is infelicitous when the Question property is N-open. However, it's possible for the question property to end up describing an open interval as long as it was also possible for the property to describe a closed interval (as long as it is not *necessarily* open).

4.5. Ruling out Contradictions

von Fintel 1993:

(78) a. Every man but John arrived.
It's not true that every man arrived, yet it is true that every man other than John arrived.
b. *A man but John arrived.
It's not true that a man arrived, yet it is true that a man other than John arrived.

Dowty 1979:

(79) *John accomplished his mission for an hour.
There is a time interval in the past T s.t. Length(T) = one hour and
$\forall t \subseteq T$ John accomplished his mission in t.

But, we seem to know what to do with certain contradictions.

(80) a. This table is both red and not red.
b. He's an idiot and he isn't.
c. #I have a female (for a) father.

Gajewski (2002, 2003): Nevertheless there is a general condition that disapproves of contradictions.[10]

There are also cases (from Barwise and Cooper) where sentences are claimed to be ruled out because they express tautologies.

(81) *There is every man in the room.
Every man is in the domain of individuals

But:

(82) a. Every man exists.
b. You're either married or you're not.

Gajewski's proposal:

A sentence is ruled out if it's LF' is either analytically true or analytically false.

An LF' of a sentence is derived from its LF by replacing every occurrence of a non-logical word by a different variable.[11]

What are the logical words?

[10] Jon pointed out to me a different approach discussed in Chierchia and McConnell-Ginet (chapter 8 section 5).
[11] If proposals such as that of Marantz (1997) and Borer (2003) are correct, LF' might be LF.

a. The words that DS mentions in its axioms (or rules of inference).
b. The words for which, as semanticists, we give informative lexical entries.
c. The function words.
d. A certain specified subset of the possible permutation invariant lexical entries[12]
d. All of the above.

Note: If DS is thought of in syntactical terms (the terms of logical syntax), then our goal would be to spell out axioms and rules of inference as possible theories of DS.

If the arguments for the UDM are correct, the following must be theorems of DS.

1. $d_1 > d_2 \rightarrow \exists d_3(d_1 > d_3 > d_2)$
2. Lexical n-place relations are upward monotone.
3. If R is a lexical n-place relation, whose m^{th} argument is a degree, then for every *w*, and for every $x_1 \ldots x_{n-1}$ $Max_{inf}(\lambda d.R(x_1)\ldots(d)\ldots(x_{n-1}))(w)$ is defined.
4. Two existential quantifiers can be commuted.

Contradictory presuppositions (Heim 1984, Lahiri 2002, Guerzoni 2003)

(83) I didn't read even one book.
Even(I didn't read ONE book)
Assertion: I didn't read one book.
Presupposition: For me not to read one book is more unlikely than all of the alternatives.

(84) *I read even one book.
Even(I read ONE book)
Assertion: I read one book.
Presupposition: For me to read one book is more unlikely than all of the alternatives.

Sentences are ungrammatical if their presuppositions are contradictory. (See Zucchi 1995.)

Could the latter principle account for some of the facts that motivating a ban an contradictions?

Possible lexical entry for *but* (following Gajewski 2002):

$but'(A_{et}) = \lambda P_{<et,t>}: \exists X_{et}.P(X)\ \&\ \forall Y_{et}.P(Y)\ (X \subseteq Y).\ P(A)\ \&\ \forall Y_{et}.P(Y)\ (A \subseteq Y).$

Gajewski's structure:

(78)' a. but({John}) λA.[Every man & $\neg$A arrived]
Presupposition, there is a minimal set A, s.t. Every man that isn't an A arrived.
Assertion: that set is the singleton set containing John
a. but({John}) λA.[A man & $\neg$A arrived]

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[12] I understand that the term goes back to Tarski. See Sher 1991 and references therein. For its use as a constraint on "grammaticalization" see von Fintel (1995).

Presupposition, there is a minimal set A, s.t. A man that isn't an A arrived.
Assertion: that set is the singleton set containing John

Similarly for our cases: we might claim that *only* and *exh* presuppose that Max_{inf} is defined for their sister.

Reasons to be suspicious:

(85) One is (only) required to weigh more than 350_F pounds to participate in this fight.
Had one only been required to weigh 300_F pounds, we could all participate.

We could add ◊ to the statement of the presupposition, but…