Implicature Calculation, *only*, and lumping: another look at the puzzle of disjunction
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1. Implicatures and the puzzle of disjunction (overall proposal)

Principles of communication allow the listener to infer (upon hearing (1) that unless the speaker believed that (1alt) were false, the speaker would have uttered (1alt).

(1) John did some of the homework.
(1alt) John did all of the homework.

Since the speaker didn’t make this alternative utterance, it follows that the speaker believes that (1alt) is false.

Derived Implicature: (S believes) it’s not the case that John did all of the homework.

1.1. The Puzzle of Disjunction.¹

(2) John did the reading or some of the homework.

By parity of reasoning: Principles of communication should allow the listener to infer (upon hearing (2) that unless the speaker believed that (2alt) were false, the speaker would have uttered (2alt):

(2alt) John did the reading or all of the homework.

Since the speaker didn’t make this alternative utterance, it should follow that the speaker believes that (2alt) is false.

Derived Implicature: (S believes) it’s not the case that John did the reading or all of the homework.

Problem: \(\neg(p\lor q) \equiv \neg p \land \neg q\); although we get the correct implicature that John didn’t do all of the homework, we also get the incorrect implicature that John didn’t do the reading (cf. Chierchia, Schwarz, and Sauerland, among others).

1.2. The proposal (in a nutshell)

- There is a systematic way to state the “scalar implicature” of a sentence explicitly: append the focus particle *only* to the sentence and place focus on scalar items.

¹ I learned about this puzzle, in its general form, from work by Chierchia, and by Schwarz. For a very interesting proposal in the neo-Gricean tradition, see Sauerland (in press). An earlier discussion can be found in Yae-Sheik Lee (who made a proposal somewhat akin to that of Sauerland’s).
(3)  John did some of the homework.
Implicature:
John only did SOME of the homework.

For all of the alternatives to 'some', d,
if the proposition that John did d of the homework is true,
then it is entailed by the proposition that John did some of the homework.

(4)  John bought three houses.
Implicature:
John only bought THREE houses.

For all of the alternatives to 'three', n,
if the proposition that John bought n houses is true
then it is entailed by the proposition that bought 3 houses.

(5)  John talked to Mary or Sue.
Implicature:
John only talked to Mary OR Sue.

For all of the alternatives to 'or', con,
if the proposition that John talked to Mary con Sue is true
then it is entailed by the proposition that John talked to Mary or Sue.

(6)  The only implicature generalization (OIG): Utterance of a sentence, S, as a default, licenses the inference/implicature that (the speaker believes) onlyS',
where S' is S with focus on scalar items.

• Exactly the same problem arises in the semantics of only:

(7)  Speaker A: John did the reading or some of the homework.
Speaker B: Is it possible that he did all of the homework.
Speaker A: No. he only did the reading or SOME of the homework.

For all of the alternatives to 'some', d,
if the proposition that John did the reading or d of the homework is true
then it is entailed by the proposition that John did the reading or some of the homework.

(8)  Question under discussion: Who is responsible for the fact that various things are missing from the kitchen, among them the ice cream and the candy?
A: I know that John ate the candy or some of the ice cream.
B: Do you think he might have eaten all of the ice cream?
A: No, he only ate the candy or SOME of the ice cream.

• There is a solution in the case of only, based on the notion of lumping developed in work by Kratzer (1988). This solution can be carried over to the problem with
implicatures if we develop a theory that captures the OIG. However, once the Kratzer amendment is added, the neo-Gricean theories of implicatures can no longer capture the OIG.

Two Possible Consequences:

1. Derive the facts in the semantics by appending a covert argument to the sentence which is akin in meaning to the particle *only* (Groenendijk and Stokhof 1984 and Krifka 1995). According to such a proposal the interpretation of scalar items does not involve “Scalar Implicatures” but is rather the result of systematic semantic ambiguity (cf. Chierchia 2000, and van Rooy 2002).

2. Revise the Neo-Gricean principles so that inferences made by hearers conform to the OIG.

Additional Advantage:

An account of a puzzle concerning implicatures and cumulative interpretations (Krifka 1998):

(9) 3 boys ate 7 apples.
   Implicature: it is not true that 4 boys ate 8 apples.

(10) a. I introduced 3 women to 7 men.
    Implicature: it is not true that I introduced 4 women to 8 men.
    b. I only introduced THREE women to SEVEN men.

Possible Arguments for Consequence 1:


(11) a. The man whose reading one book is my brother. The man whose reading two books is my brother in law.
    b. The man whose only reading ONE book is my brother. The man whose reading two books is my brother in law.

(12) a. The students who did the reading or the homework are in worse shape than the students who did both.
    b. The students who only did the reading OR the homework are in worse shape than the students who did both.

2. An Intervention effect.

(13) a. Winnie might smoke three cigarettes.
    b. Winnie is allowed to smoke three cigarettes. (Sauerland in press)
(14)  a. *Winnie only might smoke THREE cigarettes.
    b. Winnie is only allowed to smoke THREE cigarettes.
(15)  a. Winnie might smoke exactly three cigarettes.
    b. Winnie is allowed to smoke exactly three cigarettes.

2. Background

(16)  John did some of the homework.
    Standard logical rendition:
    \[ \exists x (\text{homework}(x) \land \text{John-Did}(x)) \]
    Problematic Inference:
    John didn’t do all of the homework.
(17)  John bought 3 houses.
    Standard logical rendition:
    \[ \exists x (|x|=3 \land \text{houses}(x) \land \text{John-bought}(x)) \]
    Problematic Inference:
    John didn’t buy 4 houses.
(18)  John talked to Mary or Sue.
    Standard logical rendition:
    \[ (\text{John talked to Mary}) \lor (\text{John talked to Sue}) \]
    Problematic Inference:
    John didn’t talk to Mary and Sue.

2.1. Option 1, strengthen the meaning of the relevant lexical items

(19)  John did some of the homework.
    Alternative logical rendition:
    \[ \exists x (\text{homework}(x) \land \text{John-Did}(x)) \land \\
        \neg \forall x (\text{homework}(x) \rightarrow \text{John-Did}(x)) \]
(20)  John bought 3 houses.
    Alternative logical rendition:
    \[ \exists x (|x|=3 \land \text{houses}(x) \land \text{John-bought}(x)) \land \\
        \neg \exists x (|x|>3 \land \text{houses}(x) \land \text{John-bought}(x)) \]
(21)  John talked to Mary or Sue.
    Alternative logical rendition:
    \[ [(\text{John talked to Mary}) \lor (\text{John talked to Sue})] \land \\
        \neg [(\text{John talked to Mary}) \land (\text{John talked to Sue})] \]
(22)  Standard Lexical Entries:
    a. \[ [[\text{some}]] = \lambda A. \lambda B. A \cap B \neq \emptyset \]
    b. \[ [[3]] = \lambda A. \lambda B. |A \cap B| \geq 3 \]
c. \([\text{[or]}] = \lambda p. \lambda q. p = 1 \text{ or } q = 1\).

(23) Alternative Lexical Entries:

a. \([\text{[some]}] = \lambda A. \lambda B. A \cap B \neq \emptyset \text{ and } \neg (A \subseteq B) \quad (=\text{[[some but not all]]})
   
b. \([\text{[3]}] = \lambda A. \lambda B. |A \cap B| = 3 \quad (=\text{[[exactly 3]]})
   
c. \([\text{[or]}] = \lambda p. \lambda q. p + q = 1 \quad (=\text{[[ExOR]]})

2.2. Evidence for Standard Lexical Entries

(24)a. John did some of the homework. For all I know he might have done all of it.
   
b. \(\neq\) John did some but not all of the homework. For all I know he might have done all of it.

(25)a. If John bought 3 houses, I will be very angry with him.
   
b. \(\neq\) If John bought exactly 3 houses, I will be very angry with him.

(26)a. John talked to Mary or Bill. I hope he didn’t talk to both of them.
   
b. \(\neq\) John talked to Mary or Bill but not to both. I hope he didn’t talk to both of them.

2.3. Option 2: Ambiguity

(27) 2 Lexical Entries:

a. \([\text{[some}\_\text{weak}]] = \lambda A. \lambda B. A \cap B \neq \emptyset
   \quad [\text{[some}\_\text{strong}]] = \lambda A. \lambda B. A \cap B \neq \emptyset \text{ and } \neg (A \subseteq B) \quad (=\text{[[some but not all]]})
   
b. \([\text{[3}\_\text{weak}]] = \lambda A. \lambda B. |A \cap B| \geq 3
   \quad [\text{[3}\_\text{strong}]] = \lambda A. \lambda B. |A \cap B| = 3 \quad (=\text{[[exactly 3]]})
   
c. \([\text{[or}\_\text{weak}]] = \lambda p. \lambda q. p = 1 \text{ or } q = 1.
   \quad [\text{[or}\_\text{strong}]] = \lambda p. \lambda q. p + q = 1 \quad (=\text{[[ExOR]]})

2.4. The Exhaustivity Generalization

But this is a bad proposal for two reasons:

a. it misses a generalization, and
b. it’s empirically inaccurate (downward entailing contexts)

2.4.1. The Generalization:

The phenomenon we are dealing with is pretty general, and multiplying meanings at will misses the generalization:

(28) a. It’s warm outside. (Likely inference: It is not hot outside)
   
b. If it’s warm outside, you don’t need to take a sweater.
   \(\neq\) If it’s warm but not hot outside, you don’t need to take a sweater.

(29) a. Mary is as tall as John is. (Likely inference: Mary is not taller than John is.)
   
b. Mary is as tall as John is. For all I know, she might be taller
(≠Mary is exactly as tall as John is. For all I know, she might be taller.)

(30)  
a. It’s possible that there is a sneak in the box.  
  (Likely inference: It’s not necessary…)
b. You shouldn’t open the box if it’s possible that there is a sneak inside.

(31)  
a. John started working on his experiment.  
  (Likely inference: he didn’t finish)
b. If you start working on your experiment, we will all be happy.

The generalization refers to a class of lexical entries (quantifiers, numeral expressions, truth conditional operators, comparatives, modal operators…), which are members of postulated scales, Horn Scales.²

Quantifiers: {Some, Many/Much, Most, Every/All} 
Numerals: {one, two, three,…} 
Truth conditional operators {or, and} 
Comparative operators {as, er} 
Various gradable adjectives {warm, hot}, {small, tiny} {big, huge}, etc
Modal operators {possible, necessary}

(32) The Exhaustivity Generalization: utterance of a sentence, S, as a default, licenses the inference that (the speaker believes that) all of the scalar alternatives of S that are logically stronger than S are false (Henceforth, the Exhaustivity Inference).

The Scalar Alternatives of a sentence S, Alt(S), are the set of sentences that can be derived from S by replacing scalar items in S by their scale-mates.

(33) Example: 
John bought 4 houses is a Scalar Alternative of John bought 3 houses. Since John bought 4 houses is logically stronger, The Exhaustivity Generalization tells us that utterance of John bought 3 houses, as-a-default, licenses the inference that (the speaker believes) that John didn’t buy 4 houses.

2.4.2. Lexical Ambiguities are empirically insufficient (in downward entailing contexts the relevant inferences are reversed)

(34) John didn’t do all of the homework.

Exhaustivity Inference: 
John did some of the homework.

² I represent Horn-Scales as unordered sets for reasons discussed in Sauerland (in press). In particular, the generalization needs to make reference to an ordering relation among sentences, which makes it unnecessary to order the lexical items.
2.5. The (neo)-Gricean Account (Horn, Gazdar, …)³

The Exhaustivity Inferences do not follow from the semantics of sentences but rather from pragmatic reasoning about the belief-state of speakers.

2.5.1. The short version (which doesn’t really work):

(35) John bought 3 houses.

(36) Hearer’s reasoning:
If John bought 4 houses, that would have been relevant information. S did not provide me with this information. It is therefore reasonable to assume that S thinks that John did not buy 4 houses.

2.5.2. The formal nature of the set of alternatives

Why not the following:

(37) *Hearer’s reasoning:
If John bought exactly 3 houses, that would have been relevant information. S did not provide me with this information. It is therefore reasonable to assume that S thinks that John did not buy exactly 3 houses.

If the exhaustivity inference is to follow from reasoning about the alternative utterances that the speaker avoided, something needs to be said in order to insure that we have the right set of alternatives.

(38)a. John bought 4 houses ∈ {S: Hearer considers S as a possible alternatives when hearing (35)}

b. John bought exactly 3 houses ∉ {S: Hearer considers S as a pos. alt. when hearing (35)}

Necessary stipulation: {S: Hearer considers S as a possible alternatives when hearing X} = Alt(X)

2.5.3. As it stands the hearer is only justified in making a weaker inference

(Soames 1982:455-456; Groenendijk and Stokhof 1984)

(36) Hearer’s reasoning:
If John bought 4 houses, that would have been relevant information. S did not provide me with this information. It is therefore reasonable to assume that S thinks that John did not buy 4 houses.

Wait a second. That was a little hasty. All I can conclude at the moment is that S is not in a position to claim that John bought 4 houses. The reason for this could

³ The presentation in this subsection is based on class notes of Kai von Fintel and Irene Heim.
be that S thinks that John didn’t buy 4 houses. But it could just as well be the case that S doesn’t know whether or not John bought 4 houses.

Necessary assumption (opinionated speaker): When S is uttered by a speaker s, the hearer’s default assumption is that for every member of Alt(S), s has an opinion as to whether or not S is true.

2.5.4. The long version (which does work):

Hearer’s assumptions:

1. Maxim of Quantity: speakers know that they have to make the most informative relevant contribution to a conversation.
2. Alternative Set: the set of candidates from which the most informative needs to be chosen is constructed with reference to Horn Scales; it is Alt(S).
3. Opinionated Speaker (OS): (as a default) speakers are assumed to have an opinion regarding the truth-value of Alt (S).

(39) Context: A speaker s utters the sentence, *John bought 3 houses*.

1. Given the maxim of quantity, we can infer that it’s not the case that s thinks about one of the stronger alternatives in the designated set that it is true.
2. The set of alternatives contains *John bought 4 houses*, which is logically stronger than the speaker’s utterance. Hence given 1 it’s not the case that speaker thinks that this sentence is true.
3. Given OS the default assumption is that the speaker has an opinion as to whether *John bought 4 houses* is true or false. Given 2 (the conclusion that it’s not the case that the speaker thinks that the sentences is true), we can conclude that the speaker thinks that it is false.

So we do not derive the conclusion that S is false, but only the conclusion that the speaker thinks S is false. This might be good enough. If a speaker utters a sentence and by that conveys his belief that a certain proposition, p, holds, it is natural that we will accept p whenever we accept the speakers utterance, and that p will seem to be an inference of the sentence. (However, much of the philosophical literature tries to derive something stronger, something like “mutual knowledge” of the speaker’s belief that the stronger alternatives in Alt (S) are false. One could imagine that this is necessary for explaining a no response in a TVJT paradigm that satisfies S but not its implicature.)

Standard terminology:

a. “Implicatures”: inferences from sentences based on reasoning about speakers beliefs.

b. “Scalar Implicatures”: Implicatures that rely on the Maxim of Quantity, Horn-Alternatives, and the assumption of an Opinionated-Speaker.
3. An alternative, ambiguity, account (G&S 1984; Krifka 1995)

3.1. Background: the semantics of *only* and association with focus

(40) a. Mary only introduced JOHN to Sue.
b. Mary only introduced John to SUE.
b. Mary only introduced JOHN to SUE.

LFa: only [C][VP Mary introduced John to Sue]
LFb: only [C][VP Mary introduced John to Sue]
LFc: only [C][VP Mary introduced John to Sue]

What the sentences in (40) say is that among the propositions in the set C, the only proposition that is true is the proposition that Mary introduced John to Sue. The sentences differ in the value of C something that needs to follow from the theory of focus (For discussion see Rooth (1995), Beaver and Clark (2003)).

(41) a. \[ C_{40a} = \{ p_{st}: \exists x \in D_e \text{ and } p = \lambda w. \text{Mary introduced } x \text{ to Sue in } w \}. \]
b. \[ C_{40a} = \{ p_{st}: \exists x \in D_e \text{ and } p = \lambda w. \text{Mary introduced John to } x \text{ in } w \}. \]
c. \[ C_{40a} = \{ p_{st}: \exists x, y \in D_e \text{ and } p = \lambda w. \text{Mary introduced } x \text{ to } y \text{ in } w \}. \]

(42) \( C \) is (a subset of) the focus value of VP (Foc(VP)).

(43) Informally: Foc(VP) is the set of propositions that can be derived from the interpretations of various modifications of VP; modifications in which focused constituents are replaced by various alternatives.

(44) \[ [[\text{only}]] = \lambda C_{<st,t>.} \lambda p_{st}. \lambda w: p(w) = 1. \forall q \in C. (q(w) = 1) \rightarrow (q = p) \]

3.2. A modification in the semantics of *only* \(^4\)

(45) a. John only talked to [Bill and Mary].
#That’s not true. Look, he talked to BILL.
b. John only read THREE books.
#That’s not true. Look, he read two books.

(46) \[ [[\text{only}]] = \lambda C_{<st,t>.} \lambda p_{st}. \lambda w: p(w) = 1. \forall q \in C. (q(w) = 1) \rightarrow (p \Rightarrow q) \]

(47) The alternatives of Scalar items are their scale-mates.

Consequence of this observation: the focus value of a sentence, S, in which the set of focused constituents is the set of scalar items in S is Alt(S)

a. John only read THREE books. He didn’t read FOUR.

b. John only read SOME books. He didn’t read ALL books.

a. John only talked to Mary OR Sue. He didn’t talk to Mary AND Sue.

3.3. The Postulation of a null exhaustivity operator

(48) a. John only read THREE books. He didn’t read FOUR.

b. John only read SOME books. He didn’t read ALL books.

a. John only talked to Mary OR Sue. He didn’t talk to Mary AND Sue.

3.3. The Postulation of a null exhaustivity operator

(49) Speaker A: Look at these 10 boys. Which of them do you know?
Speaker B: I know John and Bill.
Inference: B doesn’t know any of the other boys.

(50) EX(C)[I know [John and Bill]]

(51) [[EX]] = λC₁₈,₁₈,₁₈,₁₈. λpst. λw.  p(w) =1 and ∀q∈C. (q(w) =1) → (p ⇒ q)

3.4. The Exhaustivity Generalization

(32) The Exhaustivity Generalization: utterance of a sentence, S, as a default, licenses the inference that (the speaker believes that) all of the scalar alternatives of S that are logically stronger than S are false (Henceforth, the Exhaustivity Inference).

The Scalar Alternatives of a sentence S, Alt(S), are the set of sentences that can be derived from S by replacing scalar items in S by their scale-mates.

This generalization follows if, as-default, sentences are interpreted as answers to questions and the following stipulation holds:

(52) Stipulation: scalar items are inherently focused.⁶

Hope: We don’t need this stipulation. The Exhaustivity Generalization holds only when the scalar item is focused which is often enough the case. (See von Rooy 2002.)

(2) I have three children.
default structure: EX(C)[S I have threeF children].

[[(53)]] = 1 iff I have (at least) 3 children and every proposition in C that is true is entailed by the proposition that I have (at least) 3 children.
iff I have (at least) 3 children and every proposition in Alt(S) that is true is entailed by the proposition that I have (at least) 3 children.

The New Ambiguity Hypothesis: All sentences are systematically ambiguous. The source of this ambiguity is an optional exhaustivity operator.

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⁶ See Krifka (1995) for a particular implementation of the stipulation.
Chierchia’s Pragmatic Principle (cf. Dalrymple et. al 1994, 1998): When a sentence is ambiguous the default interpretation is the strongest alternative.

4. The Puzzle of Disjunction

4.1. The puzzle of multiple disjunction

(54) Sue talked to John or Mary or Bill.

The strong meaning that we want for this sentence is the proposition that Sue talked to exactly one among John, Mary and Bill. But we don’t get this meaning from any distribution of ExOr, e.g.:

(55) \( p \wedge (q \wedge r) \)

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<th>p \wedge (q \wedge r)</th>
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The exhaustivity proposal is not better off:

As we’ve seen, it works for ordinary (single) disjunction:

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7 I don’t know how far back this observation goes, but see [Reichenbach, 1947 #4892], page 45.
But it fails once embedding (multiple disjunction) is allowed:

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We get a meaning equivalent to \( \neg p \land \text{Ex}[c](q \lor r) \))

If we consider embedded EX we are back to ExOR.

4.2. The general puzzle (see Schwarz, Chierchia, Sauerland, and Gajewski\(^8\))

(56) Jon ate the broccoli or some of the soup

(56’) Strong Meaning:

| b as ss b ∨ ss b ∧ ss b ∨ as b ∧ as Ex[c](b ∨ ss) |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 |

We get a meaning equivalent to \( \neg b \land \text{Ex}[c](ss) \))

(57) Let q be a formula that has at least one occurrence of a scalar item, such that q is not the strongest element in Alt(q):

\[ \text{Ex}[c](p \lor q) \text{ is equivalent to } \neg p \land \text{Ex}[c](q) \]

Proof:

\[ \Rightarrow \]

Let q’ be an arbitrary sentence in Alt(q), which is stronger than q.

1. \[ \text{Ex}[c](p \lor q) \Rightarrow \neg (p \lor q’) \Rightarrow \neg p \]

2. Since \[ \text{Ex}[c](p \lor q) \Rightarrow p \lor q, \] given 1 we get \[ \text{Ex}[c](p \lor q) \Rightarrow q. \]

\(^8\) Chierchia claims that the puzzle is more general. See Gajewski’s discussion. If Chierchia has the right characterization, I have nothing useful to say. See Sauerland (2003) for an interesting argument against Chierchia’s conclusion based on the principle that requires presupposition maximization.
3. Since \( \neg (p \lor q') \Rightarrow \neg q' \), given 2 we get Ex\([c]\)(p \lor q) \Rightarrow q \land \neg q'\). We didn’t make any special assumptions about q', hence
Ex\([c]\)(p \lor q) \Rightarrow Ex\([c]\)(q)

\( \Leftarrow \)
Assume \( \neg p \land Ex\([c]\)(q) \) is true
1. it follows that q is true, and hence that p \lor q is true.
2. Since \( \neg p \) is true it follows that \( \neg (p \land q') \) is true for every q'.
3. Since Ex\([c]\)(q) is true, \( \forall q' \in \forall Alt(q), \) s.t. q' \Rightarrow q, \( \neg(p \lor q') \) is true.

This problem arises in the same way under the neo-Gricean Account, since the latter is designed to capture The Exhaustivity Generalization, and sentences of the form p \lor q where q is characterized as above are counter-examples to this generalization.

(32) The Exhaustivity Generalization: utterance of a sentence, S, as a default, licenses the inference that (the speaker believes that) all of the scalar alternatives of S that are logically stronger than S are false (Henceforth, the Exhaustivity Inference).

The Scalar Alternatives of a sentence S, Alt(S), are the set of sentences that can be derived from S by replacing scalar items in S by their scale-mates.

5. A Possible Solution

The problem is not a problem of implicatures per se:

(58) A: John talked to Mary or Sue or Jane.
B: Do you think he might have spoken to two of the three?
A: No, he only spoke to Mary OR Sue OR Jane.

(59) Question under discussion: Who is responsible for the fact that various things are missing from the kitchen, among them the ice cream and the candy?
A: I know that John ate the candy or some of the ice cream.
B: Do you think he might have eaten all of the ice cream?
A: No, he only ate the candy or SOME of the ice cream.

(60) A: You can eat the candy or some of the ice cream.
B: Can I eat all of the ice cream?
A: No, You can only eat the candy or SOME of the ice cream.

5.1. Background, Kratzer and the lumps of thought

Assume the following conversation takes place at the end of a day in which B painted a still life with apples and bananas.

(61) Dialogue with a lunatic:
A: What did you do today.
B: I did very little, the only thing I did was paint a still life.
A: That’s not true. Look, you painted apples.
(pointing at the apples in the still life)

Kratzer’s Intuition: If in the relevant time slice of the world w (the day of the utterance), B painted a single still life and this still life contained apples then, in w, the fact that B painted apples is part of the fact that B painted a still life.

(62) \[[\text{Only}]\](C_{st,t})(p_{st})(w) is defined iff p is true. When defined its value is 1 iff for all q \in C, s.t. q(w)=1, the fact that makes q true in w is part of the fact that makes p true in w.

(63) \[[\text{Only}]\](C_{st,t})(p_{st})(w) is defined iff p is true. When defined its value is 1 iff for all q \in C, s.t. q(w)=1, p lumps q in w.

(64) p lumps q in w, iff every situation in w in which p is true, q is true as well.

\[\text{iff } \forall s \left[ (w \geq s \text{ and } p(s)=1) \rightarrow q(s)=1 \right]^{10}\]

For the details of the ontology see Kratzer’s paper. I will focus on some results that are relevant for the discussion that follows:

We clearly want as a result is that every situation in the relevant world in which B paints a still life is a situation in which B paints an apple. Here we might use our spatiotemporal intuitions to get the right result.

Similarly, something we clearly want is that for any world, w, and propositions, p and q, if p(w)=1 and q(w)=0, p \lor q will lump p in w:

(65) Another lunatic:
A: What did John do today.
B: He did very little, the only thing he did was eat vegetables or fruit. (I don’t remember which.)
A: That’s not true. Look, he ate vegetables.

This lumping relation (p \lor q lumps p in w) follows from Kratzer’s assumption that propositions in Natural Language are persistent. (If p(s) = 1 and s' \geq s, then p(s')=1.) If there is a situation in w, s, s.t. p \lor q(s)=1, it’s a situation in which p(s)=1, since the alternative option q(s)=1, would entail, by persistence, q(w)=1, contrary to assumption.

The idea that a world yields a lumping relation among its true propositions is supported by Kratzer’s analysis of counterfactuals, which I will not go over here. However, it is important to mention that the assumption we made above about disjunction is a crucial component of Kratzer’s account of counterfactual reasoning.

Two additional ingredients we will need:

1. An assumption that (at least in run-of-the-mill cases) existential propositions do not lump universal propositions.
   In particular, if every boy eats in w, it is not the case that the proposition that some boy eats lumps the proposition that every boy eats. Take a situation in which one of the boys eats and which doesn’t contain the other boys. In this situation the proposition that some boy eats is true. However, persistence blocks this from being a situation in which the proposition that every boy eats is true.

        B: He did very little. The only thing he did was read a book.
        A: That’s not true. He read every book that was on the reading list.

2. A very similar assumption that (at least in run-of-the-mill cases) disjunctions do not lump conjunctions.

Heim’s Problem (atelic predicates):

(67) A. What were you doing today.
    B. (Not much) I was only painting a still life.
    A. #That’s not true you were also painting apples…

Assume that A’s still life contains both apples and pears. If so, there is a situation of painting a still life which is not a situation of painting the apples, hence the proposition that B was painting the apples is not a sub-part of the proposition that he was painting a still life. This suggests that the lexical entry in (63) is still too strong.

Kratzer has weakened the requirement of true alternatives to p from being identical to p or being entailed by p to being lumped by p. But this weakening seems insufficient. We need a further weakening:

Alternative (cf., Kratzer 2002):

(68) p lumps q in w iff every maximal pertinent situation s∈w, s.t. p(s)=1 is a situation in which q(s)=1.

(69) s is a pertinent situation such that p(s)=1, if there don’t exist non-empty situations s₁ and s₂, such that s₁ + s₂ = s and p(s₁)=1 but p(s₂)=0.
(70) s is a maximal situation of type A if there is no situation s' of type A, s.t. s < s'.

5.2. Lumps and the disjunction puzzle

Before taking Kratzer’s observation into account, we had the problem that Ex[c](p ∨ q) entails ¬p whenever q has a strong scalar alternative, q'. This entailment goes through because (p ∨ q') is stronger than (p ∨ q). And because, subsequently, the semantics of Ex[c](p ∨ q) excludes (p ∨ q'), hence excludes p.

But now the entailment doesn’t go through. Now Ex[c](p ∨ q) excludes p ∨ q' only in a world in which it is not lumped by p ∨ q. But when p is true and q is false, every situation in which p ∨ q is true is a situation in which p is true, hence a situation in which p ∨ q' is true. Ex[c](p ∨ q) will exclude p ∨ q' only if q is true. But if q is true we indeed want both p and q' to be false.

(56) Jon ate the broccoli or some of the soup.

LF: EX [C] [Jon ate the broccoli orF Jon ate someF of the soup].

(56) is true in w iff

a. the proposition p (= that Jon ate the b. orinc Jon ate someinc of the s.) is true in w and
b. All propositions in C that are true in w are lumped by p in w.

C = b ∨ ss b ∨ as b ∧ ss b ∧ as

<table>
<thead>
<tr>
<th>b</th>
<th>ss</th>
<th>as</th>
<th>b or ss</th>
<th>b or as</th>
<th>b and ss</th>
<th>b and as</th>
<th>EX[c](b or ss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>NL1</td>
<td>NL1</td>
<td>0</td>
</tr>
<tr>
<td>b.</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>NL1</td>
<td>NL1</td>
<td>&quot;</td>
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<tr>
<td>c.</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>L1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d.</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>NL1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>e.</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<td>1</td>
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<tr>
<td>f.</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

EX[c](b ∨ ss) is equivalent to
(b ∨ ss) ∧ ¬(b ∧ SS) ∧ (ss→¬as)

a,b:
If b(w) = 1 and ss(w) =1, the following holds (doesn’t matter whether or not as(w) =1) :
(b ∨ ss)(w) =1 (b ∧ ss)(w) =1
Since under such circumstances (b ∨ ss) doesn’t lump (b ∧ ss),
Only[C](b or ss)(w)=0

c:

If b(w) = 1 and ss(w) =0, the following holds:
(b ∨ ss)(w) =1 (b ∨ as)(w) =1 (b ∧ ss)(w) =0 (b ∧ as)(w) =0
Since in w (b ∨ ss) lumps (b ∨ as), Only[C](b or ss)(w)=1
d:
If \( b(w) = 0 \), \( ss(w) = 1 \) and \( as(w) = 1 \), the following holds:
\[(b \lor ss)(w) = 1 \quad (b \lor as)(w) = 1 \quad (b \land ss)(w) = 0 \quad (b \land as)(w) = 0\]
Here, it is easy to convince ourselves that \( b \lor ss \) does not lump \( b \lor as \). (The situations in which \( ss \) is true are identical to the situations in which \( b \lor ss \) is true and the situations in which \( as \) is true are identical to the situations in which \( b \lor as \) is true. Since \( ss \) does not lump \( as \), as we’ve seen in the previous section \( b \lor ss \) cannot lump \( b \lor as \).) Hence, \( \text{Only}[C](b \text{ or } ss)(w) = 0 \)

e:
If \( b(w) = 0 \), \( ss(w) = 1 \) and \( as(w) = 0 \), the following holds:
\[(b \lor ss)(w) = 1 \quad (b \lor as)(w) = 0 \quad (b \land ss)(w) = 0 \quad (b \land as)(w) = 0\]
Here, of course, \( \text{Only}[C](b \text{ or } ss)(w) = 1 \)

f:
If \( b(w) = 0 \) and \( ss(w) = 0 \), then \( (b \lor ss)(w) = 0 \)
\[(b \lor as)(w) = 0 \quad (b \land ss)(w) = 0 \quad (b \land as)(w) = 0\]
Here, \( \text{Only}[C](b \text{ or } ss)(w) = 0 \)

(71) \( \text{EX}(C)(p \text{ or } _F(q \text{ or } _F r)) \)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>p \lor (q \lor r)</th>
<th>p \land (q \lor r)</th>
<th>p \lor (q \land r)</th>
<th>p \land (q \land r)</th>
<th>\text{EX}[c](p \lor (q \lor r))</th>
</tr>
</thead>
</table>
| 1 | 1 | 1 | NL1 | NL1 | NL1 | 0
| 1 | 1 | 0 | NL1 | NL1 | 0 | 0
| 1 | 0 | 1 | NL1 | NL1 | 0 | 0
| 0 | 1 | 0 | 0 | 0 | 0 | 0
| 1 | 0 | 0 | 1 | 0 | 0 | 1
| 0 | 0 | 1 | 0 | 0 | 0 | 1
| 0 | 0 | 0 | 0 | 0 | 0 | 0

(59) Question under discussion: Who is responsible for the fact that various things are missing from the kitchen, among them the ice cream and the candy?
A: I know that John ate the candy or some of the ice cream.
B: Do you think he might have eaten all of the ice cream?
A: No, he only ate the candy or SOME of the ice cream.

<table>
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<tr>
<th>c</th>
<th>all</th>
<th>sm</th>
<th>c \lor sm</th>
<th>c \lor all</th>
<th>Only[c](c \lor sm)</th>
</tr>
</thead>
</table>
| 1 | 1 | 1 | 1 | NL | 1 | 0
| 1 | 0 | 1 | 1 | NL | 1 | 0
| 1 | 0 | 0 | 1 | L | 1 | 1
| 0 | 1 | 1 | 1 | NL | 1 | 0
| 0 | 0 | 1 | 1 | 0 | 0 | 1
| 0 | 0 | 0 | 0 | undefined |

We get the right result whether or not the disjunction is focused.
6. Cumulative Readings

Consider the following sentence on the cumulative interpretation:

(72) 3 boys ate 7 apples.

Scha claims that the sentence is false if 4 boys ate 8 apples (cumulatively; henceforth 4 boys cum-ate 8 apples).

Krifka (1998) argues that this is an implicature:

(73) 3 boys ate 7 apples. And, it is even possible that 4 boys ate 8 apples.

“Grice’s maxim of Quantity…will force the speaker to choose the highest numbers n, m such that the sentence n boys ate m apples is true. This is because the sentence n boys ate m apples entails sentences like n' boys ate m' apples for certain n' smaller than n and m' smaller than m.”

Is this true?
Assume that it is.
Does the reasoning derive an implicature?

Let’s see how we derive standard implicatures.

(74) Context: A speaker utters the sentence, S, John bought 3 houses.

1. Given the maxim of quantity, we can infer that there is no stronger alternative in the set of alternatives (Alt(S)) such that s thinks that it is true.

2. Alt(S) contains John bought 4 houses, which is logically stronger than the speaker’s utterance. Hence given 1 it’s not the case that speaker thinks that this sentence is true.

3. The default assumption is that the speaker has an opinion as to whether John bought 4 houses is true or false. Given 2 (the conclusion that it’s not the case that the speaker thinks that the sentence is true), we can conclude that the speaker thinks that it is false.

Now let’s try to derive this for the cumulative interpretation

(75) Context: A speaker utters the sentence, S, 3 boys ate 7 apples.

1. Given the maxim of quantity, we can infer that it’s not the case that s thinks about any of the stronger alternatives in Alt(S) that it is true.
2. Alt(S) contains 4 boys ate 8 apples, which is logically stronger than the speaker’s utterance. Hence given 1 it’s not the case that speaker thinks that this sentence is true.

Problem: As Krifka points out, 4 boys ate 8 apples is not logically stronger. Assume that each of the 4 boys eats 2 apples.

This problem would be eliminated by any theory of implicatures that derives the OIG:

(6) The only implicature generalization (OIG): Utterance of a sentence, S, as a default, licenses the inference/implicature that (the speaker believes) only $S'$, where $S'$ is $S$ with focus on scalar items.

In particular it is eliminated if implicatures are accounted for by an exhaustivity operator (EX):

EX(C)(3F boys ate 7F apples)(w) = 1 iff

a. The proposition that 3 boys cum-ate 7 apples is true in w. (I.e., there is a set of 3 boys, B, and a set of 7 apples, A, s.t. *eat'(A)(B)(w) =1)

b. For all integers n and m if the proposition that n boys cum-ate m apples is true in w, then this proposition is lumped in w by the proposition that 3 boys cum-ate 7 apples.

For any world, w, the proposition that 3 boys cum-ate 7 apples (if true in w) does not lump in w the proposition that 4 boys cum-ate 8 apples.

More generally the (neo) Gricean Scalar Implicatures (GSI) are really quite different from EX (or from anything that would be predicted the OIG). GSIs care about strength while EX doesn’t. The two look so similar only because Alt(S) -- given the nature of Horn Scales -- is totally ordered under “strength”. But various operators can in principle change the situation. If we had such operators, they could help us distinguish GSIs from EX. A cumulative operator on a verb is one such operator. Wonder if there are others.
7. A Pragmatic derivation of the OIG

Assumptions:

1. Every assertion is an answer to the question under discussion (QUД, sometimes implicit). Questions are sets of propositions, i.e. possible answers (Hamblin).

2. When a sentence, S, has scalar constituents the question is (in most cases) Alt(S).

3. Speakers must indicate when they don’t think that their assertion is a maximal true answer to the QUД.

\[ p \in \text{Max-True}(Q)(w) \text{ iff } p \in Q \text{ and } \forall q \in Q(q(w)=1 \rightarrow p \text{ lumps q in w}). \]

(76) 3 boys ate 7 apples.

QUД: how many boys cum-ate how many apples.

Hearer’s Inference: Since the speaker (s) made no indication that s/he is unable to provide a maximal true answer to the QUД, based on 3, I can conclude that s thinks that s/he is able to do so. This means that s thinks that the proposition expressed by 3 boys ate 7 apples is maximal true answer to the QUД, i.e. that it lumps all of its true scalar alternatives.

(77) Who did you see?
I saw John.
(Implícature: I didn’t see anyone else)

(78) Who did you do all day?
I painted a still life.
(Implícature: I didn’t do anything that wasn’t part of my painting of a still life)

8. Intrusive Implicatures

If Scalar implicatures require the postulation of a covert exhaustivity operator, we expect the operator to be embeddable. This suggests an approach to the phenomena of “Intrusive Implicatures” discussed quite extensively in the literature.

8.1. Negation

(79) a. John didn’t read THREE books. He read FOUR.
b. John didn’t talk to Bill OR Mary. He talked to Bill AND Mary.
b. John didn’t talk to SOME girls. He talked to ALL girls.
8.1.1. Horn’s account

(79) are instances of meta-linguistic negation:

\[ [[\text{not}_{\text{standard}}]] = \lambda p. \ p = 0 \]
\[ [[\text{not}_{\text{meta-linguistic}}]] = \lambda S. \text{linguistic-expression}. \text{It is inappropriate to utter } S. \]

(80) a. I didn’t manage to trap two monGEESE. I managed to trap two monGOOSEs.
    b. John didn’t talk to XOMsky. he talked to CHOMsky.

Horn’s arguments: ButIP (“concessive but”) is restricted to regular negation. In different languages the two lexical items are associated with different sounds (Romance, Hebrew…).

8.1.2. Horn’s arguments:

1. Meta-Linguistic negation requires focus on the culprit.

This will remain a problem for what I propose. I will be able to capture the facts only by stipulation, but I will suggest that the stipulation might be more general.

2. but as a test for meta-linguistic negation

(81) a. John didn’t read 3 books, but 4.
    c. John didn’t read 3 books, #but he read 4.
        (cf. John didn’t read 3 books, but he read 2.)

(82) a. John didn’t talk to Bill OR Mary, but to Bill AND Mary.
    b. John didn’t talk to Bill OR Mary, #but he talked to Bill AND Mary.

(83) a. John didn’t talk to XOMsky, but to CHOMsky.
    b. John didn’t talk to XOMsky, #but he talked to CHOMsky.

Horn’s conclusion: There are two types of but. ButNP can go with meta-linguistic negation. ButIP (“concessive but”) is restricted to regular negation. In different languages the two lexical items are associated with different sounds (Romance, Hebrew…).

But I’m not sure how good this argument is:

(84) a. John didn’t read exactly 3 books, but exactly 4.
    b. John didn’t read exactly 3 books, #but he read exactly 4.

8.1.3. A possible challenge for a “meta-linguistic” account

(85) a. Fred convinced me that you read not TWO books, but THREE.

b. Fred convinced me that you talked not to Bill OR Mary, but to Bill AND Mary.

(87) a. You can come to the movies with us because we didn’t buy 2 tickets, but 3.
b. John was electrocuted because he didn’t touch the red wire OR the blue wire, but both. (Kai von Fintel, pc)

(88) a. John was upset because I didn’t eat SOME of the candy but ALL of the candy.
b. John was upset because I didn’t bring TWO friends to the party as I had promised, but THREE.
c. John was upset because his kid didn’t eat the Ice-cream OR the lollipop but BOTH of them.

8.2. Various Examples based on Levinson

(89) a. Anyone who has SEVEN children is less miserable than anyone who has EIGHT.
b. #Anyone who lives in IRAQ is in less misery than anyone who lives in BAGHDAD.

(90) a. Every student who has THREE papers to write is better off than every student who has FOUR papers to write.
b. The man with TWO children near him is my brother; the man with THREE children near him is my brother in law.
c. #The man standing next to A BOY is my brother; the man standing next to BILLY is my brother in law.

(91) a. Every student who has to solve problem 1 OR problem 2 is better off than every student who has to solve problem 1 AND problem 2.
b. The person you will see talking to a boy OR a girl will be my brother; the person you will see talking to a boy AND a girl will be my brother in law.

(92) a. Every student who has to solve SOME of the problems is better off than every student who has to solve ALL of the problems.
b. The person who can solve SOME of the problems is my brother; the person who can solve ALL of the problems is my brother in law.

8.3 A constraint on disjunction (Hurford 1974)

Hurford’s Generalization: \( A \) or \( B \) is infelicitous when \( B \) entails \( A \).

(93) a. ??John is an American or a Californian.
b. ??I was born in France or Paris.

---

Hurford used this generalization to argue for a strong meaning for disjunction (ExOR):

(94)  I will apply to Cornell or UMASS or to both.

But we can extend this to other scalar items:

(95)  a.  I will read two books or three.
       b.  I will do some of the homework or all of it.

9. An Intervention Effect

Sauerland observes the following minimal pair:

(96)  a.  Winnie might smoke three cigarettes.
       b.  Winnie is allowed to smoke three cigarettes.

This would follow are derived via an exhaustivity operator:

(97)  a.  *Winnie only might smoke THREE cigarettes.
       b.  Winnie is only allowed to smoke THREE cigarettes.

A similar contrast arises in (98), Heim (2000):

(98)  a.  Winnie might smoke exactly three cigarettes.
       b.  Winnie is allowed to smoke exactly three cigarettes.

Heim (2000) points out that the contrast in (98) follows from von Fintel and Iatridou (2001) claim that epistemic modals are lethal interveners for certain dependencies.