Presupposition Projection, Trivalence and Relevance
based on work I (Danny) presented recently UCONN and UMD[1]

1. Goals

1 To investigate the ways presuppositions project from quantificational sentences in light of the predictions of certain trivalent theories of projection (see Peters 1979, Beaver and Krahmer 2001, George 2008).
2. To argue for a bivalent method of deriving the trivalent predictions (as well as possible variations on these). The method will involve a new assertability condition (Relevance, hinted at in Fox 2008).

The condition will demand that the presupposition of an atomic sentence be met to the extent that the atomic sentence is *relevant* for determining the semantic value of the matrix sentence.

2. Projection from the Nuclear Scope An Empirical Debate

(1) x has a (unique) car

(2) x has a (unique) car

(3) x has a (unique) car

(4) **Competing Empirical Claims**:

Universal Projection (Heim 1983): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes $\forall x(A(x)\rightarrow p(x))$
Existential Projection (Beaver 1992): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes $\exists x(A(x)\wedge p(x))$
Nuanced Projection (Peters, George, Chemla): A quantificational sentence of the form $Q(A)\lambda xB(x)_{p(x)}$ presupposes different things depending on various properties of Q.

3. Trivalent Predictions (one version of Nuanced Projection)

(5) :
A sentence S is assertable given a context set C only if
$\forall w\in C$ [the denotation of S in w is either 0 or 1].

1
Emmanuel Chemla, Paul Egre, Kai von Fintel, Ben George, Irene Heim, Ofra Magidor, Alejandro Pérez Carballo, Raj Singh, Benjamin Spector, Steve Yablo, and especially to Alexandre Cremers and Philippe Schlenker.

(6) Trivalent denotation of the nuclear scope in (1)a,b,c:

$$\lambda x. \begin{cases} 1 & \text{if x has a (unique) car and x drives it to school} \\ 0 & \\ \# & \text{if x has no car (or more than one car)} \end{cases}$$

(7) **Strong Kleene:**
The denotation of S in w is
(a) 1 if its denotation (in a bivalent system) would be 1 under every bivalent correction of sub-constituents.
(b) 0 if its denotation would be 0 under every bivalent correction of sub-constituents.
(c) # if neither (a) nor (b) hold

(8) a function g:X {0,1} is a *bivalent correction* of a function f:X {0,1,#} if
$\forall x[(f(x)=0 \vee f(x)=1) \rightarrow g(x)=f(x)]$

.
(1)' x has a (unique) car
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car (and no student drives his car to school)].

(2)' x has a (unique) car
Presupposes:
Either [Every student has a car (and no student drives his car to school)] or
[Some student has a car and drives it to school] or

(3)' x has a (unique) car
Presupposes:
Either [Every student has a car (and drives it to school)] or

4. experimental evidence for nuanced projection

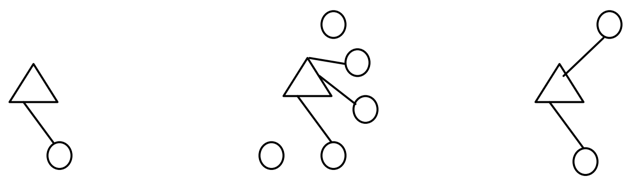
(9)
Leads only to an existential inference

(10)
Leads to a universal inference
(or at least people tend to report a universal inference more often)

These experimental results conformed with my judgments when I began thinking about it. However, it is now clear to me that there is quite a bit of variation among speakers and among presupposition triggers. In particular, it seems that in r, it7 e 151.7 Btic(bituse6()ul va)7(o4( )] e 1uupposi)- t.)

EXPERIMENT 1

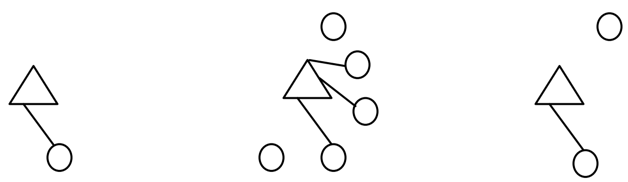
One of the following three triangles is connected to both of the circles in its vicinity.



Options: TRUE, FALSE, STRANGE

EXPERIMENT 2

None of the following three triangles is connected to both of the circles in its vicinity.



Options: TRUE, FALSE, STRANGE

5. Questions

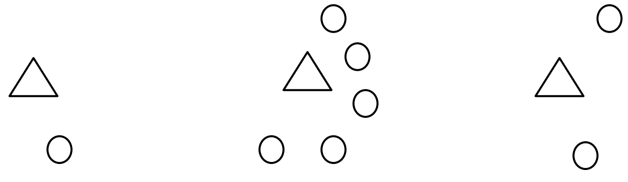
5.1. Yes/no questions

Some people who sympathize with the contrast observed by Chemla report a universal inference for yes/no questions.

(11) (Schlenker 2009)

EXPERIMENT 3

The triangles below were connected to some of the circles by lines that have been deleted. Can you help me out? Was one of the three triangles connected to both of the triangles in its vicinity?



Options: NATURAL, STRANGE

Clearly there is noise in the data (e.g. from local accommodation, introduction of the Bochval operatpr at various positions).

My hope: There will be a way to factor out this noise and then the pattern suggested by Chemla and Schlenker will emerge
data).

5.2. Constituent Questions[2]

(12) Which of these ten boys drives his car to school?
In my judgment, clearly leads to a universal inference.

My goal: To derive this universal inference from a new bivalent method of deriving the trivalent predictions.

6. More on the prediction of the trivalent presuppositions

Claim: The formal presuppositions in (1)'-(3)' do not make direct predictions for the inferences we draw from sentences. These predictions depend on our view of accommodation.

(13) QP_1 x has a (unique) car
Presupposes:
Either [QP_2 has a car and does (not) drive it to school] or
[Every student has a car] (where QP_2 can, though need not, be identical to QP_1)

Equivalently:
$\neg$[QP_2 has a car and does (not) drive it to school] $\rightarrow$
[Every student has a car]

Believing this disjunction without believing one of the disjuncts is odd. It suggests that there is a connection between the two (if one is false, the other is true). So (as in our discussion of the

contexts of use.

6.1. Indicative *some*

(1)' x has a (unique) car
Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

It is odd for a speaker to believe the disjunction without believing one of the disjuncts.

[2] Many thanks to Alexandre Cremers for drawing the relevance of constituent questions to my goals here. It will be a while before we can attempt to derive this observation.

Four scenarios to consider:

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of the common ground, C, at the point of utterance. This could be a reasonable context, but probably one in which the sentence is not assertable for Stalnakarian reasons (it is a contextual tautology).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the sentence is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation is required. By a simple-minded model of accommodation (below), accommodation is minimal leading to the C from Scenario 3. This leads to an unrealistic C.[3]

In order to deal with this, I would like to claim that the plausibility of C is investigated *only after update of the context by the assertion* (an assumption we will revisit after discussing Charlow below). The resulting C now entails the first disjunct (hence realistic).

Conclusion: there is a scenario (scenario 4) in which the sentence is acceptable without a resulting context that entails the universal statement (the second disjunct). Hence, speakers do not report a universal inference.

Presupposed Architecture:

Assertability Condition: When a sentence S is asserted in a context C it is associated with a formal presupposition p. When p is entailed by (the common ground in) C, the sentence is assertable. When p is not entailed by C, a repair strategy might come into play.

Accommodation: When p is not entailed by C, it would either be judged as unacceptable or C might be modified minimally so that p is satisfied

Accommodation (C,p) = $C \cap p$
I.e. accommodation is always minimal.

Update: After S is asserted, the context will be updated
(Update (C, S) = $C \cap \{w: S \text{ is true in } w\}$ if S is indicative)

6.2. Indicative *no*

(2)' x has a (unique) car
Presupposes:

[3] Hence, we might predict a universal inference for existential sentences of this sort. This looks like a problem, but in section 6 I will raise the possibility that it be viewed as an approach to so called strong triggers.

Either [Some student has a car and drives it to school] or
[Every student has a car]

It is very odd for a speaker to believe the disjunction without believing one of these disjuncts.

Four scenarios to consider:

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of C at the context by the assertion. In our particular case, the resulting context entails the second disjunct.

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second conjunct). Hence, speakers report a universal inference.

6.4. Negated Universals

The following suggests that a universal presupposition is wrong for universal statements:

(14) A: There are many students around, hence many cars.

Furthermore, not every student drives his car to school.
\# Furthermore, every student leaves his car at home

6.5. Yes-no Questions

(15)

Presupposes:
Either [Some student has a car and drives it to school] or
[Every student has a car]

Scenario 1: The first disjunct *some student has a car and drives it to school* is part of C at the point of utterance. This could be a reasonable context, but probably one in which the question is not assertable (the answer is already part of the common ground).

Scenario 2: The second disjunct *every student has a car* is part of C at the point of utterance. This could be a reasonable context, and one in which the question is assertable.

Scenario 3: The disjunction is part of C at the point of utterance, yet neither disjunct is. This is an unrealistic Scenario.

Scenario 4: The disjunction is not part of C at the point of utterance. Here accommodation would be required. By assumption, it is minimal and is followed by update of the context by the question. In this particular case (a question not an assertion), the resulting common ground is not affected. Since it is the unrealistic common

strengthening presuppositions).[4]

Conclusion: Under every scenario in which the sentence is acceptable, the resulting context entails the universal statement (the second disjunct). Hence, speakers report a universal inference.

[4] There are well known challenges for this line of reasoning that we will bring up in section 7 and attempt to address in section 12.

Prediction: A yes/no question will reveal weaker presuppositions if we make it plausible to believe the disjunction without believing one of the disjuncts.

(16) John and Bill meet for a game of poker. The rules they set for their engagement are the following. They each give Jane 100 dollar and get chips in return. The game will continue until one of them has no more chips left. The moment this happens, the winner (the player that has 200 chips) goes to Jane and cashes his chips.

Fred (who knows the rules of engagement) is responsible for cleaning the room the moment the game is over. He calls Jane and asks one of the following questions:

Did one of the two players cash his chips?

(17) Did anyone of these bankers acquire his fortune by wiping out one of the others?
Presupposition: if none of these bankers acquired his fortune by wiping out one of the others, they all have a fortune.

Confound (Ben George p.c.): nominals can receive temporal interpretations independent of tense, Hence it is not clear that a universal presupposition will be wrong here.

Can be addressed by explicating the temporal interpretation of the nominal:

(18) Did anyone of these bankers acquire the fortune he deposited in the bank last week by wiping out one of the others?
Presupposition: if no banker acquired the fortune he deposited in the bank last week by wiping out one of the others, they each deposited a fortune last week.

Likewise for (16):

(19) Is any one of the two players allowed to cash the chips that he now has in his possession?

7 Evidence for Universal Projection

(20) Just five of these 100 boys smoke. They all smoke Nelson
#Unfortunately, some/at least two of these 100 boys also smoke Marlboro$_F$.

onclusion: *also*
are universal.

7.1. Conflicting Data.

(21) More than 80% of the boys went to the party.
More than 40% of the boys also had a drink.
Available reading:

(22) More than 50% of Americans think that Obama is bad for America.
More than 40

(23) More than 80% of the boys went to the party.
Luckily, fewer than 50% of the boys also had a drink.

7.2. A possible reply for Charlow

(24) More than 40% of these boys also had a drink
There is a subset of salient boys who are more than 40% of the totality of boys who also had a drink.

If this is the meaning, the observation from 7.1. would still be consistent with universal projection.

(25) Possible implementation:
$[[\text{more than } 40\%\ ]]^C = \lambda A_{et}.\lambda B_{et}.|A^C \cap B| > 0.4x.|A|$
(*where A^C is the subset of A salient in C*)

Judgement?

(26) 30 of these 100 boys smoke. They all smoke Nelson
(#)Unfortunately, More than 20% of these 100 boys also smoke Marlboro_F.

7.3. A trivalent way to think about the facts

Charlow is right. We need a distinction between two types of triggers:
triggers. However, presuppositions of strong triggers do not project universally. All triggers have the disjunctive presuppositions in (1)'-(3)'.

Additional assumption: the presuppositions triggered by weak triggers can be in the scope of an Bochvar operator, which can have either embedded or matrix scope. When it has embedded scope presuppositions are cancelled. When it has matrix scope, presuppositions become part of the assertion.

Furthermore, our architecture for accommodation from section 6.1. is simply false. One must select an accommodation which is plausible *before update by the assertion*, hence a universal inference is predicted (by pragmatic strengthening) whenever the disjunctive presupposition is too weak.

Prediction: if we can make the disjunctive presupposition plausible as a minimal accommodation, there will be no universal inference.

(27) Imagine the following rather stupid game. Four players are each handed a card, and what happens next depends on whether or not one of the players gets an ace.
First possibility: No player gets an ace → Every player gets a cookie.

Second possibility: one or more player gets an ace → the player or players that get an ace get a cookie and a million dollars. No one else gets anything
So it is clear that some or all players will get a cookie.

The only reason anyone would watch the game is to find out whether someone also gets the million dollars.

(28) TV g Every week, there are ten contestants and one million dollars to be spent on prizes for the contestants. As in many TV games there are all sorts of ways of scoring points irrelevant for our issue.

Two possible outcomes
1. If everyone scores less than 1000 point, the million dollars will be used to purchase 10 diamonds (each for 100K) and each contestant will receive a diamond.
2. Otherwise, the top scoring contestant (the winner) will receive 500K and the 5 highest scoring contestants (including the winner) will each receive a (100K) diamond.

Every week at least 5 of the ten contestants get a diamond. This week one of the 10 contestant also got 500 K.

8. Challenges for the Trivalent Setup

8.1. The Proviso Challenge

The type of explanation we gave for the presuppositions of questions (4.5.) is familiar from Karttunen and Heim, and much subsequent work.

(29) a. If John is a scuba diver, he will bring his wetsuit.
Appears to presuppose: If John is a scuba diver, he has a wetsuit.
b. If John flies to London, his sister will pick him up.
Appears to presuppose: John has a sister.

The Heim/Karttunen claim: Both sentences in (29) have a conditional presupposition. It is not plausible to believe the conditional *If John flies to London, he has a sister* without believing that he has a sister. Hence, one would tend to infer that John has a sister (*pragmatic strengthening*).

Criticism by Geurts (1997): By parity of reasoning, we would expect the presupposition of (30)

(30) Bill knows that if John flies to London, he has a sister.

Conclusion reached by Singh (2008, 2010) and Schlenker (2010): if we want a mechanism that strengthens presuppositions, we need to say something that would predict when strengthening is possible.

The trivalent system would have to face the same challenge:

(31) Bill knows that either some student drives his car to school or every student has a car.

And a more specific challenge:

(32) a. Does one of your two sons drive his car to school?
b. #Does one of your two sons have a car and drive it to school or do both your sons have a car and neither drives it to school?

If trivalent presuppositions are correct, the two sentences in (32) have the same presupposition: Either (p) one of your 2 children has a car and drives it to school or (q) both of your children have a car and neither drives it to school. Furthermore, they ask for exactly the same information: they have {p,q} as their Hamblin denotation. But they feel different.

A way to approach the problem: Believing *p or q* without believing one of the disjunction is odd and thus motivates pragmatic strengthening. But such strengthening is only available in (32)a.

Note, when strengthening is not required to avoid oddity, the two questions do seem equivalent.

(33) a. Did one of the 10 bankers make his fortune by whipping out one of the others?
b. Did one of the 10 bankers make a fortune by whipping out one of the others or did they all make a fortune in some other way?

What we seem to need: a theory that would derive for each sentence the set of possible pragmatic strengthenings of its presupposition.

theory

7.3. Presupposition of non-truth-denoting expressions (thanks to A. Cremers)

The trivalent system might work for describing presupposition projection in indicative sentences which have a truth value. But how do we extend it to deal with the presupposition of non indicative sentences, e.g. questions?

Moreover, as we saw above there are interesting things to understand about non-truth denoting expressions, e.g. why *which of the boys drives his car to school* has a universal projection.

My Goal in what follows: to develop a new way of deriving the trivalent predictions in a bivalent system which will deal with the challenges mentioned in this section.

8. The setup

First Ingredient (classical bivalent semantics):

Certain lexical items will have a two dimensional entry (presupposition triggers). However semantics is not two dimensional or trivalent. Only the smallest sentences that dominate a presupposition trigger will have a two dimensional representation.

Notation: The minimal clausal node that dominates a p-trigger, S, will be annotated as S_p, where p expresses the presupposition (possibly assignment dependent). Since the system is bivalent, the semantics will behave as if p was not there.

Second Ingredient: An assertability condition

Presuppositions of complex sentences will be predicted (following Schlenker) by a pragmatic condition on an utterance of a sentence φ that has S_p as a constituent. The condition, again following Schlenker, will have a global version (that will have no left right asymmetry) that we will then incrementalize (to derive the asymmetries).

However, the pragmatic condition will be different from Schlenker

5).

$_p$ has no free variables in it (which are not in the domain of the contextually given assignment function).

9. The Propositional Case

9.1. The Global Version

Let $\varphi(S_p)$ be a sentence dominating (or identical to) S_p.

(34) $\varphi(S_p)$ is assertable in C only if
$\forall w \in C$: Relevant(S_p, $\varphi(S_p)$, w) $\rightarrow$ p is true in w.[5]

(35) Rel(S_p, $\varphi(S_p)$, w) $\Leftrightarrow_{def}$ $(([\![\varphi(T)]\!]^w \neq [\![\varphi(\bot)]\!]^w)$
Where $[\![T]\!]^w = 1$ for all w and $[\![\bot]\!]^w = 0$ for all w

9.1.1. Negation

$\varphi(S_p)$: $\neg S_p$

$\forall w\ \forall S$: Rel(S,$\neg$S, w).

Hence, $\neg S_p$ is assertable in C, by (34), only if $\forall w \in C$: p is true in w.

5 φ of S is relevant for the value of φ in w.

9.1.2. Symmetric theory of disjunction, conjunction

$\varphi(S_p)$: $S_1 \vee S_p$

$\forall w \in C$: S_1 is false in w $\rightarrow$ Rel(S_p, $\varphi(S_p)$, w).
$\forall w \in C$: S_1 is true in w $\rightarrow$ $\neg$Rel(S_p, $\varphi(S_p)$, w).

Hence $S_1 \vee S_p$ is assertable in C, by (34), only if
$\forall w \in C$: S_1 is false in w $\rightarrow$ p is true in w.

$\varphi(S_p)$: $S_1 \wedge S_p$

$\forall w \in C$: S_1 is true in w $\rightarrow$ Rel(S_p, $\varphi(S_p)$, w)
$\forall w \in C$: S_1 is false in w $\rightarrow$ $\neg$Rel(S_p, $\varphi(S_p)$, w)

Hence $S_1 \wedge S_p$ is assertable in C, by (34), only if
$\forall w \in C$: S_1 is true in w $\rightarrow$ p is true in w.

9.1.3. (Material-)Conditionals

$\varphi(S_p)$: $S_1 \rightarrow S_p$

$\forall w \in C$: S_1 is true in w $\rightarrow$ Rel(S_p, $\varphi(S_p)$, w)
$\forall w \in C$: S_1 is false in w $\rightarrow$ $\neg$Rel(S_p, $\varphi(S_p)$, w)

Hence $S_1 \rightarrow S_p$ is assertable in C, by (34), only if
$\forall w \in C$: S_1 [(1)] TJETBT/F1 12 Tf1 0 0 1 122.9 354.77 Tm[( )] T0 122.9 354.77 TTBT1 0 0 1 125.9 354.77 Tm[

φ is assertable in C only if
$\forall w \in C\ \forall a \in D_\alpha\ [Rel(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \rightarrow [\![p(x)]\!]^{w,x\rightarrow a} = 1)]$[6]

(40) $Rel(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def} \quad \exists T_a, F_a$
a. $\langle T_a,F_a\rangle$ is a-differing-extension of $S(x)_{p(x)}$ (an a-DE of $S(x)_{p(x)}$)
b. $[\![\varphi(T_a)]\!]^{w,g} \neq [\![\varphi(F_a)]\!]^{w,g}$

(41) $\langle T_a,F_a\rangle$ is an a-DE of $S(x)_{p(x)} \Leftrightarrow_{def}$
$\forall w\ [\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&$
$\forall \alpha \neq a\ [([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![F_a]\!]^{w,x\rightarrow\alpha})\ \&\ [([\![p(x)]\!]^{w,x\rightarrow\alpha} = 1) \rightarrow ([\![T_a]\!]^{w,x\rightarrow\alpha} = [\![S]\!]^{w,x\rightarrow\alpha})]]$

Equivalently:

(41)' $\langle T_a,F_a\rangle$ is an a-DE of $S(x)_{p(x)} \Leftrightarrow_{def}$
$\exists\psi\exists T_a\exists F_a$
$\forall \alpha \neq a\ [([\![p(x)]\!]^{w,x\rightarrow\alpha} = 1) \rightarrow ([\![\psi]\!]^{w,x\rightarrow\alpha} = [\![S]\!]^{w,x\rightarrow\alpha})]\ \&$
$T_a = [x = a \vee \psi]$ and $F_a = [x \neq a \wedge \psi]$

Below we state results without proofs. For proofs, see appendix B:

10.1. Binding by an expression of type e

(42) φ: John λ x has a (unique) mother
S_p x has a (unique) mother)

$\forall w \forall a[Rel(S_p, \varphi, w, a) \leftrightarrow\ a = John]$

Hence (42) presupposes that John has a unique mother.

10.2. Quantification

φ: Every(NP)(λx ($S(x)_{p(x)}$)

Claim: $\forall w \in C \forall a \in D_e$:
$Rel(S(x)_{p(x)}, \varphi, w, a) \ \leftrightarrow$
$a \in [\![NP]\!]^w\ \&\ \neg\exists b \neq a : b \in [\![NP]\!]^w\ \&\ [\![p(x)]\!]^{w,x\rightarrow b} = 1\ \&\ [\![S(x)_{p(x)}]\!]^{w,x\rightarrow b} = 0$

Hence *Every(NP)(λx ($S(x)_{p(x)}$)* presupposes that p holds of every member of the denotation of NP (the domain) or that there is one member of the domain of which p is true and S is false.

I.e., if the sentence is not false, then p must hold of every member of the domain.

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[6] φ is relevant for the value of φ in w given an individual a (or under an assignment function g, s.t. g(x)=a).

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim: $\forall w \in C \forall a \in D_e$:

$Rel(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow$

$a \in [\![NP]\!]^w$ and $\neg\exists x \neq a: x \in [\![NP]\!]^w \& \ [\![p(x)]\!]^{w, 1\to x} = 1 \& [\![S(x)_{p(x)}]\!]^{w, 1\to x} = 1$

Hence *Some(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the NP domain or that there is one member of the domain of which p holds and $[\![\lambda x S(x)_{p(x)}]\!]$ holds as well.

11. Incremental Version

(43) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if
$\forall w \in C \ \forall a \in D_\alpha \ [Rel_{inc}(S\ (x)_{p(x)}, \varphi, w, a) \to [\![p(x)]\!]^{w, x\to a} = 1]$

(44) $Rel_{inc}(S, \varphi(S), w, a) \quad \Leftrightarrow_{def} \quad \exists\varphi' \in$ GOOD-FINAL(S, φ) s.t., $Rel(S, \varphi'(S), w, a)$

More Radical Incrementalization

(45) $Rel_{r\text{-}inc}(S, \varphi(S), w, a) \ \Leftrightarrow_{def} \ \exists S'$ s.t. $Rel_{inc}(S', \varphi(S'), w, a)$

(46) φ is assertable in C only if
$\forall w \in C \ \forall a \in D_\alpha \ [Rel_{r\text{-}inc}(S\ (x)_{p(x)}, \varphi, w, a) \to [\![p(x)]\!]^{w, x\to a} = 1]$

More constituents will be r-incrementally relevant than those that are incrementally relevant (which are in turn more than those that are globally relevant). Hence, the more we incrementalize the stronger the presuppositions.

In particular, (46) will give us the Heim/Schlenker predictions (see appendix C).

12. Proviso and Formal Alternatives

: the set of possible strengthening of the presupposition of a sentence φ come from various forms of radical incrementalization, in particular by treating all sorts of constituents that do not follow the relevant presupposition trigger, as if they followed the trigger.

Since we get the classical (Heim/Schlenker) predictions by considering substitutions of the nuclear scope (which does not follow the trigger), we understand why the Heim/Schlenker presuppositions are possible strengthenings of the trivalent presuppositions.

13. Generalizing to an extensional system with any number of free variables

The global version

(47) Let $\varphi(S([x_i])_{p([xi])})$ be a sentence that dominates $S([x_i])_{p([xi])}$ where x_1 $_n$ are all the to-be-bound-variable in $S([x_i])_{p([xi])}$.

φ is assertable in C only if
$\forall w \in C\ \forall[a_i]\ Rel(S([x_i])_{p([xi])}, \varphi, w, [a_i]) \rightarrow [\![p([xi])]\!]^{w,[xi] \rightarrow [ai]} = 1)$

(48) $Rel(S([x_i])_{p([xi])}, \varphi, w, [a_i]) \Leftrightarrow_{def}$

$\exists\langle T_{[ai]}, F_{[ai]}\rangle$ s.t. $\langle T_{[ai]}, F_{[ai]}\rangle$ is an $[a_i]$-DE of $S([x_i])_{p([xi])}$ and
$[\![\varphi(T_{[ai]}))]\!]^{w,g} \neq [\![\varphi(F_{[ai]}))]\!]^{w,g}$

(49) $\langle T_{[ai]}, F_{[ai]}\rangle$ is an a-DE of $S([x_i])_{p([xi])} \Leftrightarrow_{def}$
$\forall w$
a. $\forall x \neq [ai]: [\![T_{[ai]}]\!]^{w,\,[xi] \rightarrow [ai]} = [\![F_{[ai]}]\!]^{w,\,[xi] \rightarrow [ai]}$
b. $\forall x \neq [ai]: [\![p([x_i])]\!]^{w,\,[xi] \rightarrow [ai]} = 1 \rightarrow [\![T_{[ai]}]\!]^{w,\,[xi] \rightarrow [ai]}$

]]

b. e has a job, as well.
c.

Appendices

A. More General Statements (for the propositional case of section 9)

Compositionality of Relevance (R-compositionality): Let $\varphi(S(A))$ be a sentence that dominates S which, in turn, dominates A.

a. If A is (inc-)relevant for the value of S in w, and S is (inc-)relevant for the value of φ in w, then A is (inc-)relevant for the value of φ in w.
b. If A is not (inc-)relevant for the value of S in w, A is not (inc-)relevant for the value of φ in w.
c. If S is not (inc-)relevant for the value of φ in w, A is not (inc-)relevant for the value of φ in w.

Proof: trivial.

Terminology:

If a sentence φ obeys the incremental assertability condition in (36) in every context that entails p and fails to obey the condition in every context that does not entail p, we will say that φ *presupposes p*. It will turn that for every sentence φ, there is a unique proposition that φ presupposes. Hence we can write *Presup(φ)* for this unique presupposition.

In the proofs below, we assume for simplicity that (36) is an iff condition. (It is easy to restate the proofs without this assumption.BT1 0 0 1 178.22 540.3 5.09 299.09 Tm[( )] TJET EMC /P <</MCID 18>> BD

$\forall w\in C$: Presup(φ)(w)=1.
φ is assertable in C. by definition
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (36)
$\neg\exists w\in C$, S_p dominated by $\neg\varphi$, s.t. S_p is inc-relevant for $\neg\varphi$ in w and p(w)=0.
R-compositionality

Hence $\neg\varphi$ is assertable in C.
Hence: Presup($\neg\varphi$) = Presup(φ)

A.2. disjunction

Presup($\varphi\vee\psi$) = Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ))

Proof:
Let C be a context that does not entail Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ)).
Let $w\in C$ be a world in which Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ)) is false.

First Possibility -- Presup(φ) is false in w:

$\exists S_p$ dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (36)
S_p is incrementally relevant for $\varphi \vee \psi$ in w. choose contradiction for ψ
Hence $\varphi\vee\psi$ is not assertable in C by (36).

Second Possitivlity -- ($\neg\varphi\rightarrow$Presup(ψ)) is false in :,

$\neg\varphi$ is true in w and Presup(ψ) is false in w. by (36)
Since $\neg\varphi$ is true in w, ψ is relevant for the truth value of $\varphi \vee \psi$
and the rest is just as above

Hence $\varphi\vee\psi$ is not assertable in C

Under both possibilities $\varphi\vee\psi$ is unassertable in C.

Let C be a context that entails Presup(φ)$\wedge$($\neg\varphi\rightarrow$Presup(ψ)).

$\forall w\in C$: Presup(φ)(w)=1.
φ is assertable in C. by definition
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for φ in w and p(w)=0. by (36)
$\neg\exists w\in C$, S_p dominated by φ, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and p(w)=0.
R-compositionality

$\forall w\in C$
if φ(w) = 1, ψ is irrelevant for the value of $\varphi\vee\psi$, and so is any S_p dominated by ψ
R-compositionality
if φ(w) = 0, then Presup(ψ)(w)=1 $C \Rightarrow \neg\varphi\rightarrow$Presup($\psi$)
So, there will be no S_p dominated by ψ, which is both inc. relevant for ψ and p(w)=0.
Hence:
$\neg\exists w\in C$, S_p dominated by ψ, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and p(w)=0.
R-compositionality

Hence
$\neg\exists w \in C$, S_p dominated by $\varphi\vee\psi$, s.t. S_p is inc-relevant for $\varphi\vee\psi$ in w and p(w)=0.
Hence, $\varphi\vee\psi$ is assertable in C.

Hence: $\text{Presup}(\varphi\vee\psi) = \text{Presup}(\varphi)\wedge(\neg\varphi\rightarrow\text{Presup}(\psi))$

B. Missing Proofs from section 10

B.1. Binding by an expression of type e

(53) φ: John λ x has a (unique) mother
$S(x)_{p(x)}$ x has a (unique) mother)

$\forall w \forall a[\text{Rel}(S(x)_{p(x)}, \varphi, w, a) \leftrightarrow a = John]$

Proof (trivial):
$\text{Rel}(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{John } \lambda x T_a]\!]^w \neq [\![\text{John } \lambda x F_a]\!]$ $\leftrightarrow$ by lambda conversion

$\exists\langle T_a,F_a\rangle$ $[\![T_a]\!]^{w,x\rightarrow John} \neq [\![F_a]\!]^{w,x\rightarrow John}$ $\leftrightarrow$ by definition of a-DE

$a = John$

Hence (42) presupposes that John has a unique mother.

B.2. Quantification

φ: Every(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w \in C \forall a \in D_e$:

$\text{Rel}(S(x)_{p(x),} \varphi, w, a) \leftrightarrow$

$a \in [\![NP]\!]^w \,\&\, \neg\exists b \neq a{:}b \in [\![NP]\!]^w \,\&\, [\![p(x)]\!]^{w,\, x\rightarrow b} = 1 \,\&\, [\![S(x)_{p(x)}]\!]^{w,\, x\rightarrow b} = 0$

Proof:
$\text{Rel}(S(x)_{p(x),} \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{every NP } 1T_a]\!]^w \neq [\![\text{every NP } 1F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![NP]\!]^w \subseteq [\![T_a]\!]^{w,x\to a} \wedge \neg([\![NP]\!]^w \subseteq [\![F_a]\!]^{w,x\to a})$ $\leftrightarrow$ $[\![T_a]\!]^{w,x\to a} \setminus [F_a]\!]^{w,x\to a} = \{a\}$

$a \in [\![NP]\!]^w \ \& \ \exists\langle T_a,F_a\rangle\ \forall b\neq a[\ b\in[\![NP]\!]^w \to (x\in[\![T_a]\!]^{w,x\to b})]$
$\leftrightarrow$ by definition of a-DE

$a \in [\![NP]\!]^w \ \& \ \forall b\neq a[\ b \in [\![NP]\!]^w \to ([\![p(x)]\!]^{w,\,x\to b}=0 \text{ or } [\![S(x)_{p(x)}]\!]^{w,\,x\to b}=1]]$

$\leftrightarrow$ replace $\forall$ with $\neg\exists\neg$
and let negation migrate rightwards
$a \in [\![NP]\!]^w \ \& \ \neg\exists b\neq a{:}b\in[\![NP]\!]^w \ \& \ [\![p]\!]^{w,\,x\to b}=1 \ \& \ [\![S(x)_{p(x)}\ ]\!]^{w,\,x\to b}= 0$

Hence *Every(NP)(λx (S(x)*$_{p(x)}$*)* presupposes that p holds of every member of the denotation of NP (the domain) or that there is one member of the domain of which p is true and S is false.

I.e., if the sentence is not false, then p must hold of every member of the domain.

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w\in C\forall a\in D_e$: Rel$(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$

$a \in [\![NP]\!]^w$ and
$\neg\exists x\neq a{:}x\in[\![NP]\!]^w \ \& \ [\![p(x)]\!]^{w,\,1\to x}= 1 \ \& \ [\![S(x)_{p(x)}\ ]\!]^{w,\,1\to x}= 1$

Proof:
Rel$(S(x)_{p(x)}, \varphi, w, a)$ $\leftrightarrow$ by definition of relevance

$\exists\langle T_a,F_a\rangle$ an a-DE of $S(x)_{p(x)}$ s.t.
$[\![\text{some NP } \lambda x T_a]\!]^w \neq [\![\text{some NP } \lambda x F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\varnothing\neq[\![NP]\!]^w \cap [\![T_a]\!]^{w,x\to a} \ \& \ \varnothing=[\![NP]\!]^w \cap [\![F_a]\!]^{w,x\to a}$ $\leftrightarrow$ $[\![T_a]\!]^{w,x\to a} \setminus [F_a]\!]^{w,x\to a} = \{a\}$

$a \in [\![NP]\!]^w \ \& \ \exists\langle T_a,F_a\rangle\ \forall b\neq a[\ b\in[\![NP]\!]^w \to (b\notin[\![T_a]\!]^{w,x\to b})]$
$\leftrightarrow$ by definition of a-DE

$a \in [\![NP]\!]^w \ \& \ \forall b\neq a[\ b \in [\![NP]\!]^w \to ([\![p(b)]\!]^{w,\,x\to b}=0 \text{ or } [\![S(x)_{p(x)}]\!]^{w,\,x\to b}=0]]$

$\leftrightarrow$ replace $\forall$ with $\neg\exists\neg$
and let negation migrate rightwards
$a \in [\![NP]\!]^w \ \& \ \neg\exists b\neq a{:}b\in[\![NP]\!]^w \ \& \ [\![p]\!]^{w,\,x\to b}=1 \ \& \ [\![S(x)_{p(x)}\ ]\!]^{w,\,x\to b}= 1$

Hence *Some(NP)(λx (S(x)$_{p(x)}$)* presupposes that p holds of every member of the NP domain or that there is one member of the domain of which p holds and $[\![\lambda x S(x)_{p(x)}]\!]$ holds as well.

C. Understanding the consequences of r-incrementalization

To get the Heim/Schlenker Generalization, we will strengthen the assertability condition by weakening our global notion of relevance to what we call potential-relevance (Rel_p). It will be easy to see that what we said in section 11 is correct: the incrementalization of Rel_p will be equivalent to the r-incrementalization of our earlier notion Rel.

(54) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound variable in $S(x)_{p(x)}$ (i.e. a variable free in S_p and bound in φ).

φ is assertable in C only if
$\forall w \in C\ \forall a \in D_\alpha\ (Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \rightarrow [\![p(x)]\!]^{w,x\rightarrow a} = 1)$

(55) $Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}$
$\exists T_a, F_a$
a. $[\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\rightarrow \alpha} = [\![F_a]\!]^{w,x\rightarrow \alpha})$ and
b. $[\![\varphi(T_a)]\!]^{w,g} \neq [\![\varphi(F_a)]\!]^{w,g}$

Equivalently:
(55)' $Rel_p(S(x)_{p(x)}, \varphi(S(x)_{p(x)}), w, a) \Leftrightarrow_{def}$
$\exists \psi \exists T_a \exists F_a$
a. $T_a = [x=a \vee \psi]$ and $F_a = [x \neq a \wedge \psi]$
b. $[\![\varphi(T_a)]\!]^{w} \neq [\![\varphi(F_a)]\!]^{w}$

C.1. Binding by an expression of type e

(56) φ: John λ x has a (unique) mother
$S(x)_{p(x)}$ x has a (unique) mother)

For every w:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \leftrightarrow a = John$.

Proof (trivial):
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a, F_a\ [\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\rightarrow \alpha} = [\![F_a]\!]^{w,x\rightarrow \alpha})$ &
$[\![John\ \lambda x T_a]\!]^{w} \neq [\![John\ \lambda x F_a]\!]$ $\leftrightarrow$ by lambda conversion

$\exists T_a, F_a\ [\![T_a]\!]^{w,x\rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x\rightarrow a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\rightarrow \alpha} = [\![F_a]\!]^{w,x\rightarrow \alpha})$ &

$[\![T_a]\!]^{w,x\to John} \neq [\![F_a]\!]^{w,x\to John}$ $\Leftrightarrow$

$a = John$

Hence (56) presupposes that John has a unique mother.

C.2. Quantification

φ: Every(NP)(x (S(x)$_{p(x)}$)

Claim:

$\forall w \in C \forall a \in D_e$:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \Leftrightarrow a \in [\![NP]\!]^w$

Proof:
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a, F_a\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\to \alpha} = [\![F_a]\!]^{w,\ x\to \alpha})\ \&$
$[\![\text{every NP } \lambda x T_a]\!]^w \neq [\![\text{every NP } \lambda x F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists T_a, F_a\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\to \alpha} = [\![F_a]\!]^{w,\ x\to \alpha})\ \&$
$[\![NP]\!]^w \subseteq [\![T_a]\!]^{w,x\to a} \wedge \neg([\![NP]\!]^w \subseteq [\![F_a]\!]^{w,x\to a})$ $\leftrightarrow$

$a \in [\![NP]\!]^w$

Hence *Every(NP)(λx(S(x)$_{p(x)}$)* presupposes that p holds of every member of the denotation of NP

φ: Some(NP)(λx (S(x)$_{p(x)}$)

Claim:

$\forall w \in C \forall a \in D_e$:

$Rel_p\ (S(x)_{p(x)}, \varphi, a, w) \leftrightarrow a \in [\![NP]\!]^w$

Proof:
$Rel_p\ (S(x)_{p(x)}, \varphi, a, w)$ $\leftrightarrow$ by definition of p-relevance

$\exists T_a, F_a\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\to \alpha} = [\![F_a]\!]^{w,\ x\to \alpha})\ \&$
$[\![\text{some NP } \lambda x T_a]\!]^w \neq [\![\text{some NP } \lambda x F_a]\!]^w$ $\leftrightarrow$ lambda conversion + the observation that $F_a \subset T_a$

$\exists T_a, F_a\ [\![T_a]\!]^{w,x\to a} = 1\ \&\ [\![F_a]\!]^{w,x\to a} = 0\ \&\ \forall \alpha \neq a\ ([\![T_a]\!]^{w,x\to \alpha} = [\![F_a]\!]^{w,\ x\to \alpha})\ \&$
$[\![NP]\!]^w \cap [\![T_a]\!]^{w,x\to a} \neq \emptyset$ and $[\![NP]\!]^w \cap [\![F_a]\!]^{w,x\to a} = \emptyset$ $\leftrightarrow$

$a \in [\![NP]\!]^{w}$

Hence *Some(NP)(λx(S(x)*$_{p(x)}$*)* presupposes that p holds of every member of the NP domain.

D. Problem from Infinite Domains

(57) An infinite number of boys drove their car to school.
$[_{\varphi}$ An infinite number of boys $[_{S(x)}$ $_{\text{x has a unique car}}]$

$\forall w \in C\ \neg\exists a \in D_e\ (Rel(S(x), \varphi, w, a))$.
Hence the sentence should presuppose nothing.

Revision:

(58) Let $\varphi(S(x)_{p(x)})$ be a sentence that dominates $S(x)_{p(x)}$ where x is a variable of type α, the single to-be-bound-variable in $S(x)_{p(x)}$

φ is assertable in C only if
$\forall w \in C\ \forall A \subseteq D_\alpha\ S(x)_{p(x)}\ \ Rel_{SUB\text{-}SET}\ (S(x), \varphi, w, A) \rightarrow \exists A' \subseteq A (\forall a \in A' [\![p(x)]\!]^{w,x \rightarrow a} = 1)$[7]

Equivalently: φ is assertable in C only if
$\forall w \in C\ \forall A \subseteq D_\alpha\ S(x)_{p(x)}\ \ Rel_{SUB\text{-}SET}\ (S(x), \varphi, w, A) \rightarrow \exists a \in A ([\![p(x)]\!]^{w,x \rightarrow a} = 1)$

(59) $Rel_{SUB\text{-}SET}\ (S(x), \varphi, w, A)\ \ \Leftrightarrow_{def}$
$\exists T_A, F_A$
a. $\langle T_A, F_A \rangle$ is an *A*-differing-extension of $S(x)_{p(x)}$ (an *A*-DE of $S(x)_{p(x)}$)
b. $[\![\varphi(T_A)]\!]^{w,g} \neq [\![\varphi(F_A)]\!]^{w,g}$

(60) $\langle T_A, F_A \rangle$ is an *A*-DE of $S(x)_{p(x)}$ if
$\forall w\ \forall a \in A [\![T_A]\!]^{w,x \rightarrow a} = 1\ \&\ [\![F_a]\!]^{w,x \rightarrow a} = 0\ \&$
$\forall \alpha \notin A\ [([\![T_a]\!]^{w,x \rightarrow \alpha} = [\![F_a]\!]^{w,\, x \rightarrow \alpha})\ \&$
$[([\![p(x)]\!]^{w,x \rightarrow \alpha} = 1) \rightarrow ([\![T_a]\!]^{w,x \rightarrow \alpha} = [\![S]\!]^{w,\, x \rightarrow \alpha})]]$

Note: this assertability condition is stronger than what we had previously since:

a. $\forall S, \varphi, w, a [Rel(S(x), \varphi, w, a) \rightarrow Rel_{SUB\text{-}SET}\ (S(x), \varphi, w, \{a\})]$
b. If $|A| = \infty\ \exists S, \varphi, w\ [Rel_{SUB\text{-}SET}\ (S(x), \varphi, w, A)\ \& \forall a \in A \neg Rel\ (S(x), \varphi, w, a)]$

E. More General Statement (for a language with variables)

Hopefully some other time

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7 $_{SUB\text{-}SET}(S(x), \varphi$ hould be read as the value of S(x) is relevant for the value of φ in w for some subset of A.