Negative Islands and Maximization Failure*

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Goals:

1. To quickly review the proposal of Fox and Hackl (2006) for the sensitivity of degree constructions to NIs, and the ramifications of this proposal for the architecture of grammar.
2. To present Abrusán and Spector's observation (2008, in press) that F&H's account is not general enough: degree question are ambiguous and F&H can only deal with one of their readings.
3. To point out, along the lines of Fox (2007), that the essential piece of A&S's proposal is not in conflict with F&H.
4. To present evidence from degree relative clauses that A&S's account cannot replace F&H's (although I will also point out a possible interesting consequence unrelated to NIs).
5. To entertain a completely different account of A&S's ambiguity, relating it to a general ambiguity of questions, identified in Spector (2007, 2009).
6. To attempt an account for the NI observed in Spector (2007, 2009) in terms of Maximization Failure.

1. Fox and Hackl (2006)

(1) a. How many books is john required to read?
What is the number, n, such that John is required to read (at least) n books?
i.e., what is the number n that leads to the most informative true proposition of the form John is required to read n books?

b. How fast is John required to drive?
What is the degree, d, such that John is required to drive (at least) d fast?
i.e., what is the degree d that leads to the most informative true proposition of the form John is required to read d books?

Often Postulated:

A. Scales (various domains of degrees/numbers)
B. A notion of measurement: functions from various objects to various scales
C. Two types of scales, and hence two different notions of measurement:
   1. Various expressions that combine with count nouns (*3 girls, how many boys, more than three girls*) require the notion of cardinality/counting for their understanding, i.e. a function from sets to discrete scales.
   2. Other degree expressions (*6 feet tall, how tall, taller than 6 feet, how much water*) rely on different scales, perhaps on dense scales, something closer to the real or rational numbers.

Goal of F&H: To argue that degree/measurement scales are always dense:

* Thanks to Márta Abrusán, Micha Breakstone, and especially to Benjamin Spector.

(2) The Universal Density of Measurements (UDM): Measurement Scales that are needed for Natural Language Semantics are *always* dense.

In other words, to argue for two claims:

(3) a. The Intuitive Claim: Scales of height, size, speed, and the like are dense.
b. The Radical Claim: All Scales are dense; cardinality in not a concept of NLS.

The argument is based on an account for NI in degree questions and definite descriptions, and on a related constraint on implicatures and corresponding sentences with *only*. I will focus on our discussion of NI in questions, though at certain points I will allude to the other accounts as well.

1.1. Density as an intuitive property of scales

THE BASIC EFFECT

(4) *How much does John not weigh? (Obenauer, Rizzi., Rullmann, passim)
What is the (immediate) successor of John's weight?
(the minimal amount d, such that he doesn't weigh d much)?

Given the UDM, no degree has a(n immediate) successor; hence the question can have no answer.

More specifically: Questions denote sets of possible answers (of the sort described in Hamblin 1973). And, a question demands that one of the possible answers be more informative than all true alternatives (cf. Dayal 1996).

(5) **Maximality Requirement**: The (Hamblin 1973) denotation of a question, Q, must have a member which is true and logically stronger than all other true members of Q.

In the case of (4), the Maximality Requirement can't be met given the UDM:

If John weighs 150 pounds, the set of true answers to (4) is

$\{\lambda w.\neg$ John weighs d (or more) pounds in w: $d > 150\}$.

The members of this set are logically stronger the smaller d is, and, given the UDM, there is no strongest member: since there is no minimum in $\{d: d>150\}$.

Similar explanation to the following:

(6) a. John weighs more than 150 pounds.
*Implicature: There is no degree great than 150, d, s.t. John weighs d pounds.
I.e., John weighs exactly S(150) pounds. Where S is the successor function
b. John weighs very little. *He only weighs more than 150_F.

EXISTENTIAL MODALS BELOW NEGATION CIRCUMVENT THE PROBLEM

(7) a. How much radiation are we not allowed to expose our workers to?
cf. **How much radiation did we not expose our workers to?*
b. How much money are we not allowed to bring into this country?

Explanation: Although there can be no minimal degree, d, s.t. we did not expose our workers to d much radiation (given the UDM), there can be a minimal degree, d, s.t. we are not allowed to expose our workers to d much radiation (despite the UDM).

Demonstration: If the only rule states that we should not expose our worker to d_{min} radiation (or to more than that).

Similar explanation for the following:

(8) a. You're required to weigh more than 300 pounds (if you want to participate in this fight).
Implicature: There is no degree greater than 300, d, s.t. you are required to weigh more than d pounds.
b. You're only required to weigh more than 300 pounds.

UNIVERSAL MODALS BELOW NEGATION DO NOT

(9) a. How much money are we not allowed to bring in to this country?
b. *How much money are we not required to bring in to this country?

(10)a. How much radiation is the company not allowed to expose its workers to?
b. *How much food is the company not required to give its workers?[†]

Explanation: Assume there was a minimal degree, d, such that we are not required to expose our workers to d much radiation. Call it d_{min}. It fallows that there is an allowed world w, such that we did not expose our workers to d_{min} radiation in w. Well, how much radiation did we expose our workers to in w? Call this amount d_r. d_r must be the successor of d_{min}, in contradiction to density.

Similar explanation for the following:

(11)a. You're allowed to weigh more than 150 pounds (and still participate in this fight).
*Implicature: There is no degree greater than 150, d, s.t. you are allowed to weigh more than d pounds.
b. *You're only allowed to weigh more than 150 pounds.

[†] Ignore the following irrelevant reading: What is the amount of food such that there is food in that amount and the company is not required to give that food to its workers. To avoid this problem:
(i) (When you enter the country) How much money are you not allowed to have.
(ii) *(When you enter the country) How much money are you not required to have.

1.2. Density as a formal property of scales

THE BASIC EFFECT

(12) *How many kids do you not have?

EXISTENTIAL MODALS BELOW NEGATION CIRCUMVENT THE PROBLEM

(13) a. If you live in China, how many children are you not allowed to have?
b. How many days a week are you not allowed to work (according to union regulations)?
c. How many soldiers is it (absolutely) certain that the enemy doesn't have?

UNIVERSAL MODALS BELOW NEGATION DO NOT

(14) a. *If you live in Sweden, how many children are you not required to have?
b. *How many days a week are you not required to work (even according to the company's regulations)?
c. *How many soldiers is it possible that the enemy doesn't have?

(15) a. ???Combien John n'a-t-il pas lu de livres?
How many John n'has-he not read of books
b. ? Combien peux-tu me dire avec (absolue) certitude que John n'a pas lu de livres?
How many can-you me tell with (absolute) certainty that John has not read of books
(Benjamin Spector, pc)

(16) a. *Combien Jean n'a-t-il (pas) d'enfants?
How many John n'has-he not of children
b. ?Combien les chinois ne peuvent ils (pas) avoir d'enfants?
How many the chineese n'alloweed-them not have of-children?
(Valentine Hacquard, pc)

1.3. Further evidence for the account (if we only had time)

-Definite Descriptions, Implicatures, *Only* and Density (F&H)

-Negation and Density (Nouwen 2008)

1.4. Relevance for Architecture

If this account is right, the Maximality Requirement must be computed before *contextual parameters are introduced* to derive truth conditions. In particular, before a contextually given level of precision breaks dense domains into discrete equivalence classes. (See F&H for details.)

Hence, if this account is right, we might want to think of the inference between syntax and semantics as an interface level between language and logic: between syntax and an internal Deductive System (See Chierchia 1984, Fox 2000, Gajewski 2002, 2008, F&H, for related ideas and various forms of evidence).

1.5. Other Cases of Maximization Failure

In Fox (2007), I suggested a generalization of F&H:

In particular, restricting our attention to questions, assume that Q has the following properties:

1. $\exists p \in Q$ (p is non-contradictory)
2. $\forall p \in Q$(it is logically impossible for p to be the most informative true member of Q)

a. Q will suffer from Maximization Failure (MF): it will necessarily violate the Maximality Requirement, and will thus be ruled out.

b. Modal Obviation: Easy to prove that
    1. $\{\Box p: p \in Q\}$ will not suffer from MF.
    2. $\{\Diamond p: p \in Q\}$ will still suffer from MF.

This observation could be responsible for similar effects to those observed by F&H, in cases where density seems to be irrelevant:

-Implicatures, Only and Symmetry (Spector 2006, Fox 2007)

-Manner Questions and Symmetry (Abrusán 2007, 2008)

As I pointed out in Fox (2007), however, the observation of the more general pattern might motivate us to re-examine the sources of MF in the basic cases: is density really the culprit or could there be alternative sources?

I argued that for *only* and *SIs* density is indeed the culprit (an argument that in my view is strengthened quite a bit by Nouwen 2008).

Here, I would like to discuss an alternative to density for NIs proposed by Abrusán and Spector, and to argue, again, that density, although not the whole story, is a crucial component.

2. Abrusán and Spector's Challenge

Consider the rules described by the following:

(17) Question: How fast are we required to drive on this highway?
Answer 1: 40 miles an hour.
Answer 2: Between 40 and 70 miles an hour.

The two possible answers seem to reflect a genuine ambiguity:

(18) a. John already knows how fast we're required to drive on this highway. What he would like to know now is the maximum allowed speed.
b. John doesn't know how fast we're required to drive on this highway. He knows that the minimum allowed speed is 40 miles an hour but he doesn't know that the maximal allowed speed is 70 miles an hour.

The UDM account of NI was based on a relatively standard semantics for degree expressions, which can only capture the first meaning. It is not obvious that the account will be preserved, once room is made for the second reading.

My Goal: To find a way to preserve the UDM account while making room for the ambiguity.

In more detail:
(a) To argue that the UDM account is correct for Reading 1
(b) To present two conflicting possibilities for Reading 2
Either:
(1) A&S's account of it is correct or
(2) it is an instance of a more general non-basic (type shifted) reading of questions which occurs outside the domain of degree questions, and is also sensitive to NIs.
(c) Pursuing (2), to entertain an account for the sensitivity of the more general type shifted reading to NIs, also in terms of Maximization Failure.

3. A&S's Proposal

3.1. The Account of the Ambiguity (building on Heim 2006):

(19) Reading 1: What is the maximally informative interval, I, which contains the maximal degree, d, such that it is required that we drive at least d fast on this highway?

Equivalent to: What is the singleton set {d} such that d = Max({d: it is required that we drive at least d fast on this highway})?

Reading 2: What is the maximally informative interval I, such that it is required that I contain the maximal degree, d, such that we drive at least d fast on this highway?

Equivalent to: What is the smallest interval I such that it is required that our driving speed be in I?

3.2. A&S's account of the negative island

(20) *Guess how tall John is not?

Reading 1: What is the maximally informative interval I, which contains the maximal degree, d, such that it is not the case that John is d tall (or taller)?

Ruled out for reasons discussed by Rullmann (1995), building on von Stechow (1984): there is no maximal degree, d, such that it is not the case that John is d tall (or taller).

Reading 2: What is the maximally informative interval I, such that it is not the case that I contains the maximal degree d such that John is d tall (or taller).

Ruled out as a case of MF: there can be no maximally informative interval of the relevant sort; the intervals become more informative the larger they are, and there is no maximal interval that fails to contain John's height.

Modal obviation is predicted for reading 2, based on the logic of Fox (2007).

4. Minimal Degree vs. Maximal Interval

Consider a case of modal obviation such as the following:

(7)b How much money are we not allowed to bring into this country?
Answer under F&H: d_{min}
I.e.: the minimal amount, d, such that we are not allowed to bring d-much money into the country.
Answer under A&S: $[d_{min}, \infty)$, when defined (see section 5)
I.e.: the maximal *interval,l I, such that I contains no amount of money that we're allowed to bring into this country.*

Definite descriptions teach us that at least sometimes d_{min} is right.

4.1. Definite Descriptions are like questions (setting the stage)

(21) a. *The amount of money that you don't have with you is $10000.
b. The amount of money that you are not allowed to have with you when you enter this country is $10000.
c. *The amount of money that you are not required to have with you when you enter this country is $10000.

The pattern seen in (21) is exactly parallel to what we see in questions. A&S did not talk about definite descriptions, so I'd like to compare what I think they could say to what we say in F&H.

My conclusion is that A&S can account for the pattern of acceptability, but not for the meaning of the sentences, at least not in any obvious way.

Assumption about the meaning of *the* (without which, I don't see how to get off the ground):

(22) $[[\text{the}]](P_{\alpha,st})(w) = (\iota x_\alpha)(P(x)(w) = 1 \text{ and } \forall y\ [P(y)(w) \rightarrow P(x) \text{ entails } P(y)])$
(Evidence from von Fintel, Fox, and Iatridou, partially reviewed in F&H)

Under F&H: The definite description in (b) refers to d_{min}.

Under A&S: The definite description in (21)b should refer to the entire interval of amounts of money that you're not allowed to bring into this country: $[d_{min}, \infty)$.

4.2 The Argument for d_{min}

is $10000 is true of d_{min} but not of $[d_{min}, \infty)$, at least not in any obvious way.

Way out for A&S #1:

(23) **Minimum Predication:** A predicate of degrees, P, when applied to an interval I must be type shifted. The shif yields a predicate that is true of an interval I if P is true of Min(I).

But I don't think Minimum Predication is true in the general case.

Let's start with the basic ambiguity that A&S identified. First I don't think there is an ambiguity with definite descriptions:

(24) a. Reading 1 (degree):
The speed at which you're required to drive is 30 miles an hour.

b. Reading 2 (interval):
?? The speed at which you're required to drive is any speed between 30 and 70 miles an hour.[‡]

So this is a problem. But even if I'm wrong here's an additional problem:

(25) a. The speed at which you're required to drive – 30 miles an hour – is very low.
b. ?? The speed at which you're required to drive – any speed between 30 and 70 miles an hour – is very low.

To the extent that (25)b is good (which, again, I do not think it is), it entails that every speed in the interval is low. This suggests that Minimum Predication is false, and, therefore, that the definite description in (21)b does not refer to the interval $[d_{min}, \infty)$.

[‡] I, myself, need the plural for Reading 2. As mentioned, this judgment, if correct, is, in-and-of-itself, an argument in favor of the minimal degree reading (more in 4.5.).

Way out for A&S # 2: restricting Minimum Predication to intervals of the form [d, ∞).

But this (a) seems arbitrary, and (b) is empirically just as problematic:

(26) a. The amount of money that you are not allowed to bring into this country, $2000, is very low.
b. #The amount of money that you are not allowed to bring into this country, any amount above $2000, is very low.

4.3. What this suggests

1. Definite descriptions of the sort we looked at must refer to degrees rather than intervals.
2. In their reference to degrees, definite descriptions are sensitive to NIs.
3. This sensitivity is subject to modal obviations, and in such cases the reference is the *minimal* degree that a predicate is true of. The explanation of MF when there is no modal obviation must, therefore, rely on density rather than on interval semantics.

4.4. Possible Modification of A&S

Questions are ambiguous between a degree and an interval semantics (Reading 1 and Reading 2, respectively). When they receive the degree semantics, the UDM is responsible for the pattern of Maximization Failure. When they receive the interval semantics, A&S's account yields precisely the same pattern, due to the logic of MF.

There is a very simple theory of the ambiguity. The covert operatory that A&S postulate to shift from properties of degrees to properties of intervals is simply optional.

4.5. Why are definite descriptions limited to d-min (in obviation contexts)?

Hypothesis: The "interval semantics" is in general not available for definite descriptions:

(27) Question: How fast are we required to drive on this highway?
Answer 1: 40 miles an hour.
Answer 2: Between 40 and 70 miles an hour.

(28) a. The required speed is 40 miles an hour.
a. The required speed is between 40 and 70 miles an hour.

(28)b suggests ignorance on the part of the speaker. In other words, the definite description does not seem to be able to denote an interval. (Cf. *The allowed range is between 40 and 70 miles an hour*.)

Similarly:

(29) a. The speed at which were required to drive is 40 miles an hour.
a. The speed at which were required to drive is between 40 and 70 miles an hour.

Possible Explanation: A construction from which a degree operator moves can shift to a property of intervals (e.g., by merging Heim's covert operator PI to the *wh*-operator), or to a property of a higher type (see section 5). But this property will not be able to combine with an ordinary degree head noun (*speed*, *amount*), because such a noun denotes a property of degrees, not of intervals.

4.6. Nouwen's Puzzle (very tentative and in conflict with the proposal in section 5)

(30) The minimal number of points we need to score in order to win the game is 6.

The puzzle: There is no minimal (other than the smallest existing degree) to the number of points we need to score.

Observation: There is a minimum to the (most informative) interval such that it is required that the number of points we score belong to that interval.

So perhaps interval semantics can help us out.

But we saw in the previous section that interval semantics is not available inside relative clauses.

Tentative Proposal: *minimal* solves the problem. It is a function that takes a predicate of intervals, and returns a predicate of degrees that can combine with the head noun.

(31) $[[\text{minimal}]] (\Re_{\langle dt,t\rangle}) = \lambda d.\ \exists Q\in\Re[d = \text{Min}(Q)]$
Where: $\text{Min}(Q) = (\iota d)(\forall d'\in Q[d\neq d'\rightarrow d<d'])$

(32) Syntax (reverse matching):
The ~~number of points~~
minimal (number of point) λD.we need PI(D)(λd. we score ~~d number of points~~)
The λd[d a number of points & d$\in$minimal(λD. $\Box$ (The number we score $\in$D))]

If we intensionalize when needed (IFA) and apply the entry for *the* in (22), we get the right results (or at least that's what I got when I computed):
The number of points which is the minimum of the smallest interval that in every allowed world contains the number of points we score.

But…

5. Problem for Intervals

Consider, again, the rules described by the following:

(33) Question: How fast are we not allowed to drive on this highway?
Possible Answer: Faster than 70 miles an hour.

Wrong prediction of A&S: In this scenario, in contrast to a case with no minimum allowed speed, there should be Maximization Failure.

Explanation:

Reading 1: What is the maximally informative interval, I, which contains the maximal degree, d, such that we are not allowed to drive d fast on this highway?

Ruled out in general: there can be no maximal degree, d, such that it is not allowed that we drive d fast on this highway.

Reading 2: What is the maximally informative interval I, such that it is not allowed that I contain our speed?

Ruled out in this particular scenario: there is no maximally informative interval of the relevant sort; the intervals become more informative the larger they are, and in this particular scenario, there is no largest interval that fails to contain an allowed speed.

A&S propose a solution for this in terms of scale truncation, but this solution will not allow them to understand the fact that (33) is ambiguous (by their own tests):

(34) Question: How fast are we not allowed to drive on this highway?
Answer 1: Faster than 70 miles an hour.
Answer 2: Faster than 70 miles an hour or slower than 40 miles an hour.

(35) John and Bill disagree on how fast we are not allowed to drive on this highway. John thinks that 30 miles an hour is below the minimum allowed, and Bill doesn't.

So it seems to me that there are two conclusions we should draw:

(36) a. Reading 1 quantifies over degrees, and its most informative answer is a minimal degree rather than a maximal one. Without this assumption we wouldn't understand answer 1 for (34), as well as the reference of definite descriptions described in the previous subsection.
b. Reading 2 quantifies over a bigger domain than the domain of intervals. This is needed to license Answer 2 for (34), and to account for (35).

Similarly:
(37) Due to Micha Breakstone (p.c.): To achieve its mission, a fighter jet must either be flown below 100 feet to avoid radar or above 3 miles to avoid missiles.
Question: At what height are we required to fly the plane (if we want to return home safely)?
Answer: Either below 100 feet or above 3 miles.

Further Evidence against Intervals:

(38) How many players do you need for the perfect bridge club?
A multiple of 4, so that everyone can play.
(For similar examples, see Beck and Rullmann 1999)

What we've learned:

1. To understand how Reading 1 is subject to NIs (at least for definite descriptions, but probably for questions too) we seem to need the UDM, since the most informative object under this reading is the *minimal* degree, not the maximal interval.

2. To understand how Reading 2 is subject to NI, we, unfortunately, cannot rely on intervals, since the most informative object provided in Answer 2 for (34)-(35) is not an interval. We also can't rely on the UDM. We just need a third account, hopefully, still in terms of MF.

6. Towards an Alternative

Reading 1 is the basic reading which involves quantification over degrees (or reference to degrees in the case of definite descriptions). This reading is subject to NI, given the UDM.

Reading 2 is a non-basic (higher type) interpretation of the question, observed for simpler questions in Spector (2007). Such higher type interpretations are also subject to NI, as pointed out in Spector (2007).

Whatever accounts for Spector's observation should be able to complete F&H's account of the NI.

Hope: Spector's observation could be accounted for in terms of MF as well, leading to a more uniform account of NI.

7. Spector (2007, 2008)

7.1. A new reading for simple *wh*-questions

Imagine that the teacher gives you the following assignment: read The Bothers Karamazov, War and Peace, and three books of your choice by an American writer, whichever three you might like. You can then say the following:

(39) a. I know which books we are required to read.
The Bothers Karamazov, War and Peace, and any three books by an American writer.
b. Bill doesn't know which books we are required to read. Although he knows we are required to read BK and WP, he does not know that we are required to read 3 books by an American writer.

(40) Spector's Ambiguity:

Reading 1: **LF-x** Which books
$\lambda x_e.\ \Box$ (we read x)?
Reading 2: **LF-Q** Which Upward Monotone GQ ranging over books
$\lambda Q_{et,t}.\ \Box$ (Q [λx. we read x])?

For the sentences in (39), the LF that is chosen is LF-Q, under which the answer to the question is:

(41) $BK \wedge WP \wedge$(any three books by an American writer)
i.e. $\lambda P.(BK \in P \wedge WP \in P \wedge P$ contains three books by an American writer)

7.2. Sensitivity to NI

(42) Q: Which books are we required to read?
A: The French books or the Russian books.
Can mean: $\Box$(We read FB or RB).

(43) Q: Which book did we not read?
A: The French books or the Russian books.
Cannot mean: $\neg$(We read FB or RB).

Spector's proposal: For the fragment answer in (43), the unavailable (narrow scope) reading requires ellipsis of the predicate *λQ. ¬(Q(λx. we read x))*. Such ellipsis is possible only if the question receives LF-Q, but this LF is subject to NI.

7.3. Selective Modal Obviation (still need to confirm judgments here, but that was the feeling when Benjamin presented this at MIT)

(44) Q: Which books are we not allowed to read?
A: The French books or the Russian books.
Can mean: $\neg\Diamond$(We read FB or RB)

Q: Which of these dishes are we not allowed to order?
A: those that contain trans fat or those with more than 2000 calories.
Can mean: $\neg\Diamond$(TF or more than 2000 calories)

(45) Q: Which books are we not required to read?
A: The French books or the Russian books.
Cannot mean: $\neg\Box$(We read FB or RB)

Q: Which of these dishes are we not required to order?
A: those that contain protein or those with fiber.
Cannot mean: $\neg\Box$(protein or fiber)

8. An account of Spector's Observation

The Maximality Requirement is satisfied in the bad question answer pairs we've looked at, so it needs to be strengthened.

(46) Universal Maximality Requirement (UMR): For every $p \in Q$, it must be possible for p to be the most informative true member of Q.

Motivation: The denotation of the question is the set of candidate answers, and it must be possible for every one of them to fulfill this role.

With this at hand, the facts about negative islands follow from the logic I discussed in Fox (2007).

The Basic Effect

In a simple question with negation, the UMR is not satisfied.

To see this, consider the GQ *both Mary and Sue* and the potential answer to the question based on this GQ: the proposition $\lambda w.\neg$ *[Both Mary and Sue*$\lambda xP(x)(w)]$.

This proposition cannot be the strongest proposition in the denotation of the question, i.e. of the form: $\lambda w.\neg [Q\lambda xP(x)(w)]$.

The reason is obvious. Assume that the potential answer is true. It follows that P is false of either Mary or Sue. So either $\lambda w.\neg$ *Mary* $\lambda xP(x)(w)$ is true, or $\lambda w.\neg$ *Sue* $\lambda xP(x)(w)$ is true and neither is entailed by the potential answer.

Pattern of Obviation follows the familiar logic:

8.1. Under Universal Modals the readings are licensed

When a universal modal is present, the UMR is satisfied.

To see this, let Q be an arbitrary upward monotone GQ and P a property (of type <e,st>). Now assume that the modal base for $\Box$ is the following:

$MB=\{w: Q\lambda x(P(x)(w)\}$.

It is easy to see that with this modal base:

a. The potential answer: $\Box[\lambda w'.Q\lambda x(P(x)(w')]$ is true
b. Any proposition of the form $\Box[\lambda w'.Q'\lambda x(P(x)(w')]$ will be true only if it entailed by the potential answer in a.

Proof:
(a) is trivial.
For (b) assume that $\Box[\lambda w'.Q'\lambda x(P(x)(w')]$ is true. It follows that $\forall w \in MB\ [Q'\lambda x(P(x)(w)]$ is true. In other words $MB(=\{w: Q\lambda x(P(x)(w)\}) \subseteq \{w: Q'\lambda x(P(x)(w)\}$. And since $\Box$ is upward monotone, $\{w: \Box^{w}\lambda w'\, Q\lambda x(P(x)(w')\}) \subseteq \{w: \Box^{w} Q'\lambda x(P(x)(w')\}$.

8.3. An existential modal below negation circumvents the problem

When negation is followed by an existential modal, the UMR is satisfied.

To see this, let Q be an upward monotone GQ and P a predicate (of type <e,st>). Now assume that the modal base for $\Diamond$ is the following:

$\{w: \neg Q\lambda x(P(x)(w)\}$.

It is easy to see that with this modal base:

a. The potential answer: $\neg\Diamond\ [\lambda w'.Q\lambda x(P(x)(w')]$ is true
b. Any proposition of the form $\neg\Diamond\ [\lambda w'.Q'\lambda x(P(x)(w')]$ will be true only if it entailed by the potential answer in a.

Proof:
(a) is trivial.
(b) reduces to our proof above given that $\neg\Diamond$ is equivalent to $\Box\neg$.
Assume that $\neg\Diamond[\lambda w'.Q'\lambda x(P(x)(w')]$ is true. It follows that $\Box\neg[\lambda w'.Q'\lambda x(P(x)(w')]$ is true: $\forall w\in MB[\neg Q'\lambda x(P(x)(w)]$.
In other words $MB(=\{w: [\neg Q\lambda x(P(x)(w)]\})\subseteq\{w: [\neg Q'\lambda x(P(x)(w)]\}$. And since $\Box$ is upward monotone,
$\{w: \Box^w\lambda w'\neg Q\lambda x(P(x)(w')\})\subseteq\{w: \Box^w\lambda w'\neg Q'\lambda x(P(x)(w')\}$, i.e.,
$\{w:[\neg\Diamond^w Q\lambda x(P(x)(w)]\})\subseteq\{w:[\neg\Diamond^w Q'\lambda x(P(x)(w)]\}$

8.4. A universal modal below negation does not circumvent the problem

To see this, consider the GQ *both Mary and Sue* and the potential answer
$\lambda w.\neg\ \Box^w\ \lambda w'$*Both Mary and Sue*$\lambda x(P(x)(w')]$.
This proposition cannot be the strongest proposition of the form
$\lambda w.\neg\ \Box^w\ \lambda Q\lambda x(P(x)(w')]$.

The reason is obvious. Assume that the potential answer is true. It follows that there is a world in MB in which P is either false of Mary or False Sue, call such a world w^0. Assume (with no loss of generality that) P is false of Mary in w^0. Then $\lambda w.\neg$ *Mary* $\lambda x(P(x)(w)]$ is true in w^0, and since w^0 is in the modal base (in the actual world) $\lambda w.\neg\ \Box^w$ $\lambda w'$*Both Mary and Sue*$\lambda x(P(x)(w')]$ is true (in the actual world).

9. Back to Degree Questions

(17) Question: How fast are we required to drive on this highway?
Answer 1: 40 miles an hour.
Answer 2: Between 40 and 70 miles an hour.

Two interpretations:

(17)' LF-d What degree
$\lambda d.\ \Box$ Our speed $\geq$ d
LF-Q What UM-GQ ranging over degrees
$\lambda Q_{dt,t}.\ \Box\ (Q\ \lambda d.$ Our speed $=$ d])?

Two additional interpretations, which will not change the empirical landscape:

(17)" LF-d What degree
$\lambda d.\ \square$ (Our speed = d)
LF-Q What UM-GQ ranging over degrees
$\lambda Q_{dt,t}.\ \square$ (Q λd. Our speed $\geq$ d])?

In An NI environment, Maximization Failure follows in two different ways depending on the choice of LF:
For LF-d, MF follows from the UDM, as discussed in F&H (though now under a stronger UMR).
For LF-Q, MF follows along lines discussed in section 8.

Unresolved Questions (stated somewhat cryptically)

a. What accounts for A&S's observations about the contextual restrictions on the use of what they call quasi negative islands?
b. What accounts for the sensitivity of Spector's reading to factive islands? (Although I did not go over this, I think the sensitivity of degree questions to factive islands follows, under both readings, from the logic of presupposition projection outlined in Abrusán (2007, 2008))?
c. What accounts for the obviation of negative islands by *wa* marking in Japanese (Shimoyama and Schwarz 2010)?

My hope is that the true answers to these questions will not affect the conclusion I would like to draw.

Conclusions:

1. The second reading A&S identified involves some form of type of shifting of the standard semantics assumed in F&H (either Heim's PI, or some other shifter of the sort that would yield the reading identified in Spector (2007, 2009)).
2. The basic reading of degree questions, however, involves simple quantification over degrees.
3. MF is the general account of NI (probably with the exceptions of *why* questions, as noted in Fox 07).
4. MF follows in degree questions from the logic of dense domains.
5. MF follows in Spector's reading from the existence of potential answers that are too weak to serve as complete answers (thus leading to a UMR violation).
6. MF in A&S second reading either reduces to 5 or follows from their own account, but then we still have to understand higher type answers that do not seem to involve intervals.